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The perfect composite index does not exist (but we have to use it)

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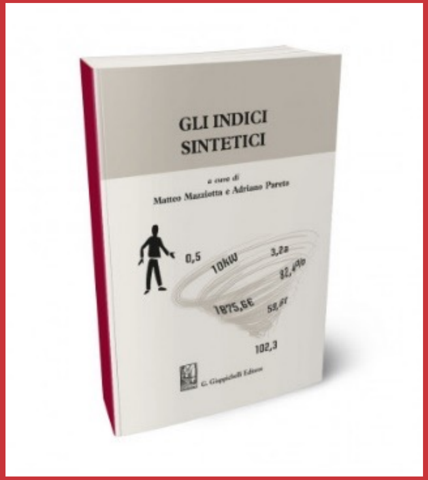


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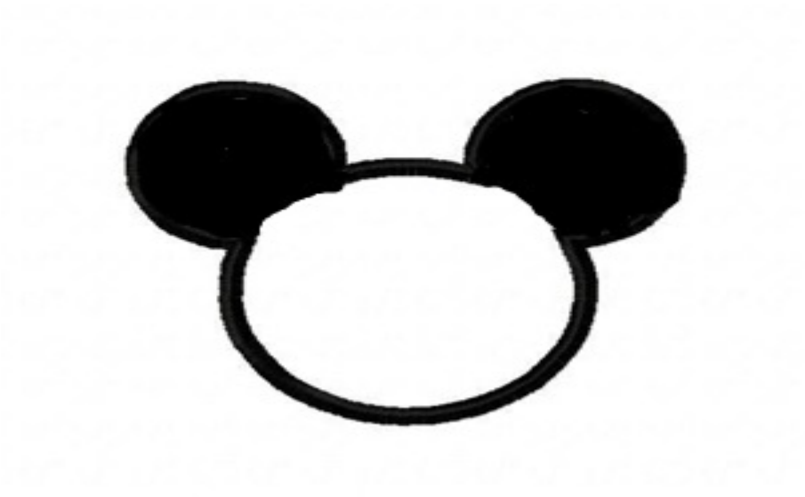
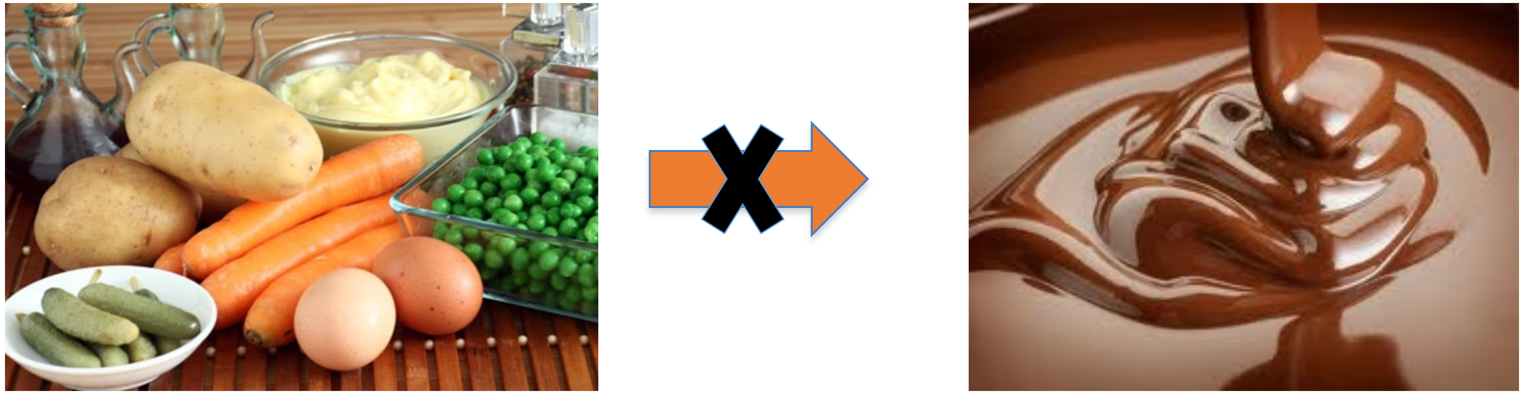
Representation of a latent factor



The quality of the statistics

From raw materials…

From raw materials…

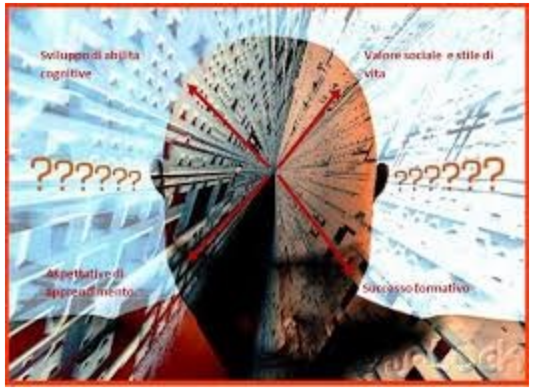


Is it possible to get chocolate from the ingredients of russian salad?

Work tools…

…are not all equal

The real issue is…



…manage the complexity.

All components of the process should be integrated with each other organically.

They can not be chosen independently

Raw materials Vs Mixture

The complexity is present in both cases

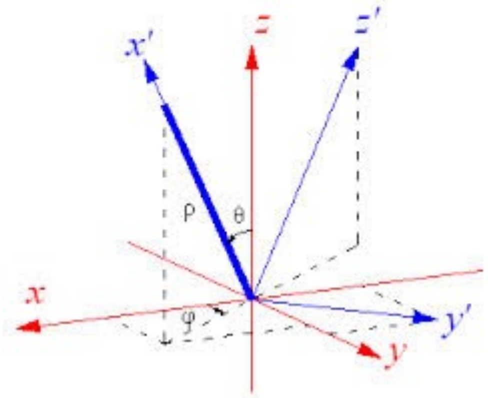
Dashboard Vs Composite index

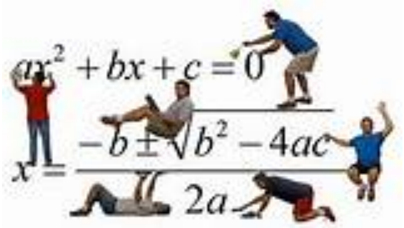
The human eye can not see more than three dimensions

Goal: reduce the dimensionality

Solution:

- Reduce the number of indicators
- Apply a composite index





Composite index

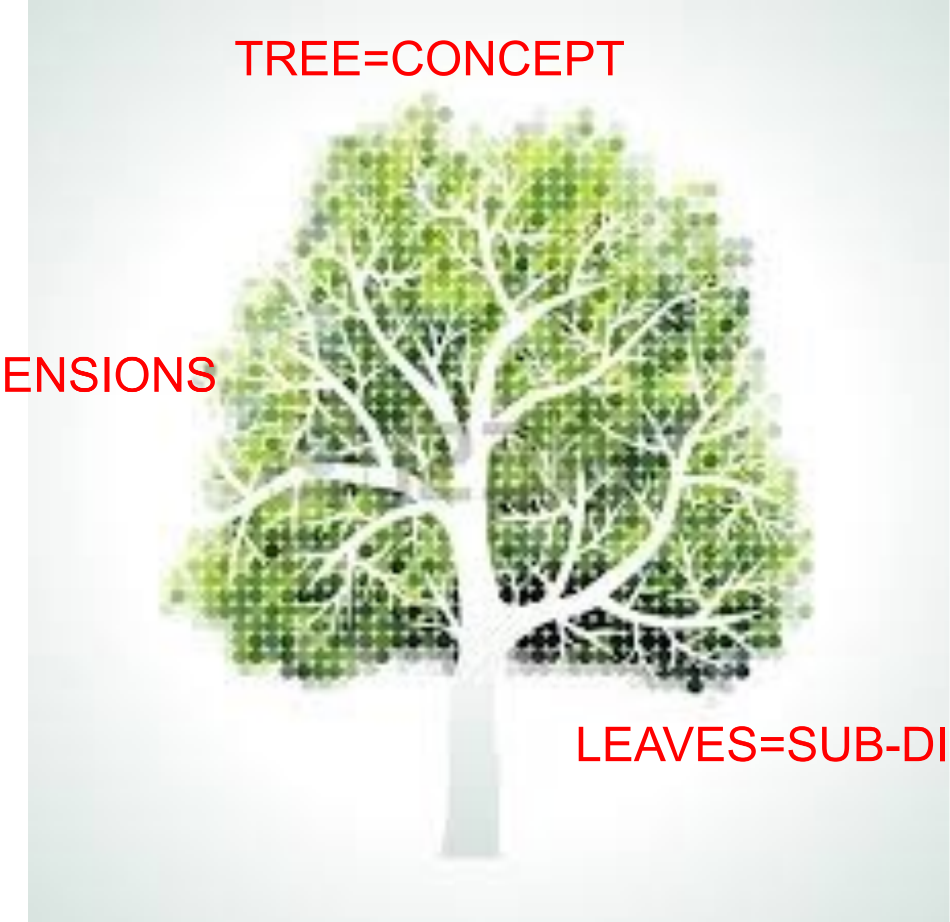
Definition

A composite index is a combination of individual indicators in a single measures based on models.

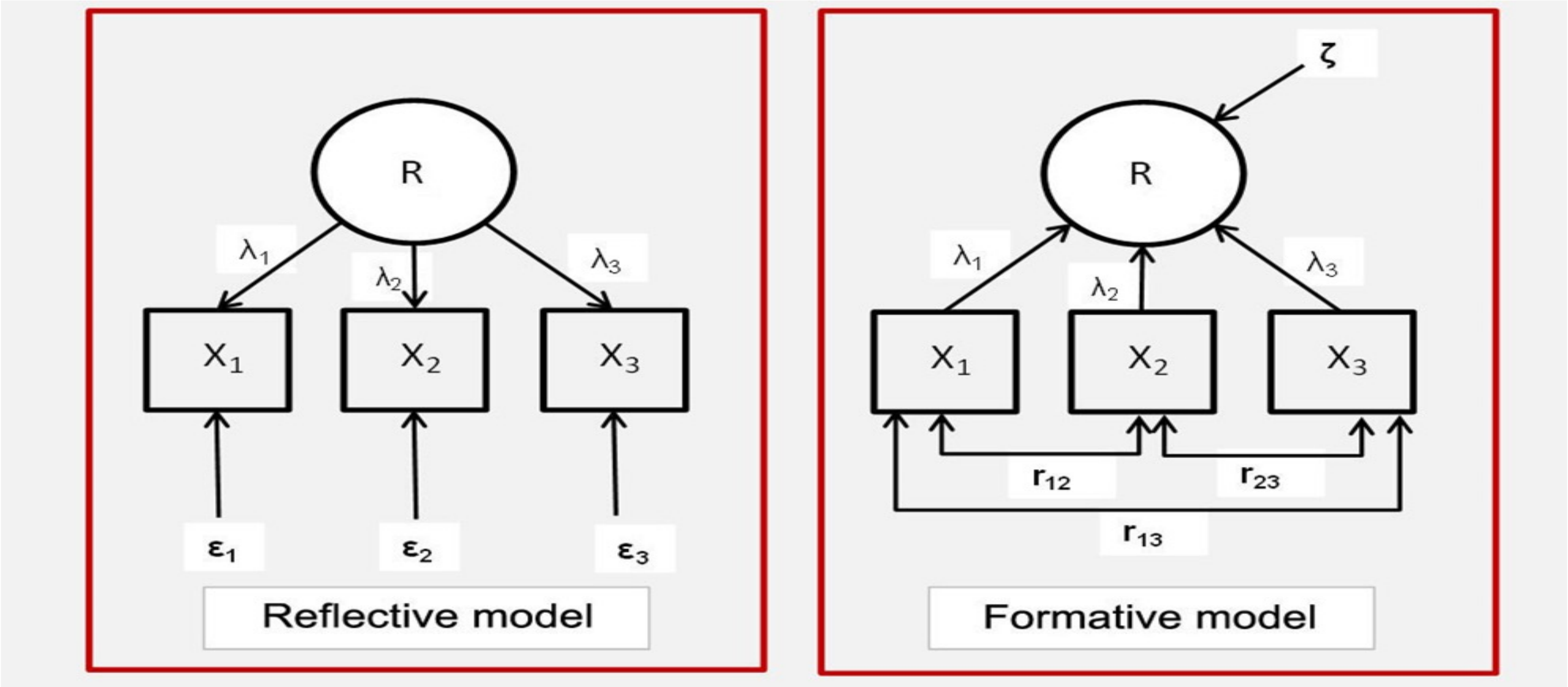
The composite index should ideally measure multidimensional concepts that can not be captured by a single individual indicator.

10 steps for constructing composite indices

1. Theoretical framework
2. Data selection
3. Imputation of missing data
4. Multivariate Analysis
5. Normalisation of individual indicators
6. Weighting and aggregation
7. Uncertainty and sensitivity analysis
8. Back to the data
9. Links to other indicators
10. Visualisation of the results


TREE=CONCEPT
BRANCHES=DIMENSIONS
LEAVES=SUB-DIMENSIONS

Reflective Vs Formative Model



Pros and Cons

PROS

- Can summarise complex, multi-dimesional realities with a view to supporting *decision-makers*;
- Are easier to interpret than a battery of many separate indicators (dashboard);
- Can assess progress of countries over time;
- Reduce the visible size of a set of indicators without dropping the underlying information base;
- Place issues of country performance and progress at the centre of the policy arena;
- Facilitate comunication with general public (i.e. citizens, media, etc.);
- Help to construct narratives for lay and literate audiences;
- Enable users to compare complex dimensions effectively.

Pros and Cons

CONS

- May send misleading policy messages if poorly constructed or misinterpreted;
- May invite simplistic policy conclusions;
- May be misured, e.g. to support a desired policy, if the construction process is not transparent and/or lacks sound statistical or conceptual principles;
- The selection of indicators and weights could be the subject of political dispute;
- May disguise serious failings in some dimensions and increase the difficulty of identifying proper remedial action, if the construction process is not transparent;
- May lead to inapprpriate policies if dimensions of performance that are difficult to measure are ignored.

Composite index

Formalization of the problem

You want to pass from the matrix $X_{n,m}$ to the vector S:

$$X_{n,m} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1j} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2j} & \dots & x_{2m} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{i1} & x_{i2} & \dots & x_{ij} & \dots & x_{im} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nj} & \dots & x_{nm} \end{pmatrix} \Rightarrow S = \begin{pmatrix} s_1 \\ s_2 \\ \dots \\ s_i \\ \dots \\ s_n \end{pmatrix}$$

where:

n is the number of the considered statistical units (i.e. areas)

m is the number of the individual indicators

x_{ij} is the value of j-th indicator in the i-th unit

s_i is the value of the composite index in the i-th unit

Composite index

Aim of the normalization

The individual indicators, in order to be aggregated, must be express in the same unit of measure and 'move' in the same direction of the studied phenomenon.

Normalization is the process for releasing the individual indicators from the unit of measure and make them concordant with the phenomenon to be measured

The main normalization methods transform the individual indicators in:

- **ranks**
- **indices based on the range** (min-max)
- **standardized values** (*z*-score)
- index **numbers**
- **percentage values**

Methods of normalization

Ranks

It replaces the value assumed by each unit, with the order number (rank) by which the unit is placed on the list according to the indicator
In formulas, you pass by x_{ij} to g_{ij}:

$$g_{ij} = \text{rank}\{x_{ij}\}$$

where $\text{rank}\{x_{ij}\}$ is the rank of the unit in the list corresponding to the indicator j.
If two or more units assume the same value, then they will give the average rank of the positions that they would have had in case of different values.
The transformation into ranks purifies the indicators from the unit of measure, but it does not preserve the relative distance between the different units.

Methods of normalization

Relative indices with respect to range (min-max)

The value assumed by each unit oscillates between the lowest value assumed by the indicator, set equal to 0, and the highest, set equal to 1.

In formulas, we transform x_{ij} in r_{ij}:

$$r_{ij} = \frac{x_{ij} - \min_i \{x_{ij}\}}{\max_i \{x_{ij}\} - \min_i \{x_{ij}\}}$$

Where $\min_i \{x_{ij}\}$ and $\max_i \{x_{ij}\}$ are, respectively, the minimum and the maximum of the individual indicator j

Through this transformation the individual indicators are purified from the unit of measure and reported in a scale from 0 to 1

Methods of normalization

Standardized values (*z*-score)

The variables are transormed in standardized values

You pass from x_{ij} to z_{ij}:

$$z_{ij} = \frac{x_{ij} - \overline{x}_j}{\sigma_j}$$

where

$$\overline{x}_j = \frac{\sum_{i=1}^{n} x_{ij}}{n} \qquad \sigma_j = \sqrt{\frac{\sum_{i=1}^{n} \left(x_{ij} - \overline{x}_j\right)^2}{n}}$$

This normalization allows to obtain values with mean equal to 0 and standard error equal to 1

Methods of normalization

Index numbers

The value assumed by each unit is divided by a reference value belonging to the same distribution or calculated on it (generally the mean or maximum)

You pass from x_{ij} to I_{ij}:

$$I_{ij} = \frac{x_{ij}}{x^*_{oj}} 100$$

where $x^*_{oj} = \frac{\sum_{i=1}^{n} x_{ij}}{n}$ or $x^*_{oj} = \max_i \left\{ x_{ij} \right\}$

This normalization allows to delete the unit of measure and to keep the relative distance among the units.

If the denominator is the maximum than you obtain values less or equal to 100.

Methods of normalization

Percentage values

The value of each unit is divided by the sum of the values

You pass from x_{ij} to p_{ij}:

$$p_{ij} = \frac{x_{ij}}{x^{*}_{oj}} 100$$

where

$$x^{*}_{oj} = \sum_{i=1}^{n} x_{ij}$$

The sum of the normalized values are equal to 100

Pros and Cons

Comparison between normalization methods

Normalization	Pros	Cons
Ranking	Insensitivity to outliers	Assumes the same distance between items
R - MinMax	Values in the range 0-1	Depends on outliers
Z - Score	Mean=0 e S.q.m.=Standard error=1	Presence of negative values
I - Index Numbers	Preserves the original variability	Strong sensitivity to outliers
P - Percentage Values	Values in the range 0-100	Assumes additivity of the phenomenon

The polarity

The polarity of an indicator is the sign of the relationship between the indicator and the phenomenon to be measured.

For example, in the calculation of a development index, GDP has positive polarity (+), while the infant mortality rate has negative (-).

Similarly, in the calculation of a poverty index, GDP negative (-), while the infant mortality rate has positive polarity (+).

In order to aggregate correctly a set of indicators, it is necessary that all have positive polarity.

The main methods of reversing the polarity is divided into:

- **Linear transformation**
- **Non linear transformation**

Methods to invert polarity

Linear transformation

It's the easiest way to reverse the polarity of an indicator and it is based on subtracting from the maximum value the value of each unit :

$$x'_{ij} = \max_i \{ x_{ij} \} - x_{ij}$$

Where $\max_i \{ x_{ij} \}$ is the maximum value of the j indicator.

This operation does not alter the distances between the different statistical units.

It is used primarily in the normalizations:

- Ranking
- Indices based on the range (min-max)
- Z-score

Methods to invert polarity

Non linear transformation

It consists in calculating the reciprocal of the value of each unit :

$$x'_{ij} = \frac{1}{x_{ij}}$$

Where x'_{ij} is the value assumed by the indicator j in the unit i.

This operation allows you to avoid the presence of null values in the distribution of the indicator normalized, but it alters the distances between the different statistical units.

It can be used in the normalization:

- index numbers
- percentage values

Weighting and aggregation

Weighting and aggregation allow:

• define a system of 'weights' in order to weigh the individual indicators in function of their different importance in describing the phenomenon;

• combine the indicators, in order to build one or more composite indices (pillars)

Not always weighting and aggregation are two distinct stages of the synthesis of the indicators.

Some methods allow you to define the system of 'weights' and aggregate individual indicators simultaneously (for example, Principal Component Analysis).

The choice of the weights

The weighting system is based on two possible approachs:

• ***Subjective approach.*** The weights are assigned on the basis of a arbitrary judgment by experts (eg. Human Development Index). In this case, the system of weights does not depend on the observed values;

•***Objective approach.*** The weights are calculated mathematically through the application of particular statistical methods (eg. Principal Components Analysis). In this case, the system of the weights depends on the observed values.

NOTE

The subjective choice to assign equal weights to the indicators elementary is the simplest solution, but not 'neutral' or without critics

Methods of aggregation

The aggregation is a technique used to reduce the multiplicity of indicators (synthesis).

The procedures can be technically simple, based on the use of mathematical functions, or more complex that may require the use of multivariate analysis:

- ***Aggregation Functions***
  - compensative approach: linear functions (i.e.. Additive methods)
  - non-compensative approach : funzioni non linear functions (i.e. moltiplicative methods)
- ***Methods of aggregation***
  - compensative approach (i.e. *PCA*)
  - non-compensative approach (i.e. *Multicriteria Analysis*)

Methods of aggregation

Arithmetic Mean

The mean is equal to:

$$ {}_{M_1}S_i = \frac{\sum_{j=1}^{m} y_{ij}}{m} $$

Where y_{ij} is the normalized value of the indicator *j* and the unit *i*.

The weighted mean is:

$$ {}_{M_1}S_i = \frac{\sum_{j=1}^{m} y_{ij} w_j}{\sum_{j=1}^{m} w_j} $$

where w_j is the weight of the indicator *j*.

Methods of aggregation

Geometric Mean

The mean is equal to:

$$ {}_{M_0}S_i = \left(\prod_{j=1}^{m} y_{ij} \right)^{\frac{1}{m}} $$

where y_{ij} is the normalized value of the indicator *j* and the unit *i*.

The weighted mean is:

$$ {}_{M_0}S_i = \left(\prod_{j=1}^{m} y_{ij}^{w_j} \right)^{\frac{1}{\sum_{j=1}^{m} w_j}} $$

Where w_j it he weight of the indicator *j*.

Methods of aggregation

Squared mean

The mean is equal to:

$$ {}_{M_2}S_i = \sqrt{\frac{\sum_{j=1}^{m} y_{ij}^2}{m}} $$

Where y_{ij} is the normalized value of the indicator *j* and the unit *i*.

La weighted mean is:

$$ {}_{M_2}S_i = \sqrt{\frac{\sum_{j=1}^{m} y_{ij}^2 w_j}{\sum_{j=1}^{m} w_j}} $$

Where w_j is the weight of the indicator *j*.

Methods of aggregation

Power mean of order r

It is a generalized mean that includes the preavius means.

The mean is equal to:

$$ {}_{M_r}S_i = \sqrt[r]{\frac{\sum_{j=1}^{m} y_{ij}^{r}}{m}} $$

and the weighted mean is:

$$ {}_{M_r}S_i = \sqrt[r]{\frac{\sum_{j=1}^{m} y_{ij}^{r} w_j}{\sum_{j=1}^{m} w_j}} $$

For r =1 we have the arithmetic mean, for r =2 we have the squared mean and for r→0 we have the geometric mean.

Example

Computation of the power means (simple and weighted)

Indicatore	**Y (valori stand.)**	**W (pesi)**	**YW**	**Y^W**	**Y²W**
I1	100,0	0,15	15,0	2,00	1500,0
I2	120,0	0,30	36,0	4,20	4320,0
I3	50,0	0,15	7,5	1,80	375,0
I4	75,0	0,20	15,0	2,37	1125,0
I5	105,0	0,20	21,0	2,54	2205,0
M1	**90,0**		**94,5**		
M0	**86,1**			**90,75**	
M2	**93,3**				**97,6**

In general, we have: $M_0 \leq M_1 \leq M_2$

Aggregation functions

Mazziotta-Pareto Index (MPI)

A composite index to measure a multidimensional phenomenon is proposed; the method is based on the hypotesis that the components are not substitutable.

This involves the introduction of a 'penalty' for the units that do not present the balanced values of the indicators.

Index requirements

- Independence from the variability of the individual indicators and from the unit of measure;
- Independence from an ideal unit, since the definition of a set of values «objective» is subjective, is not unique and it can vary over time;
- Semplicity of calculation;
- Easy interpretation.

Aggregation functions

Mazziotta-Pareto Index (MPI)

Normalization

Each individual indicators is transformed in a z-score with mean equal to 100 and standard error equal to 10 (mean=100 and s.e.=10); the normalized values will be in the range 70-130.

This procedure allows to depurate the indicators both from the unit of measure and from the variability and it does not require the deifnition of a target values (ideal unit), because it replaces the vector with the of average values.

In this way, it is easy to identify the units with a level of the phenomenon above average (values greater than 100) and the units with a level below the average (values less than 100).

Example

Comparison between normalization methods

Unità	Indicatori			Numeri indici (b=media)				Var. standardizzate			
	X1	X2	X3	I1	I2	I3	Media	Z1	Z2	Z3	Media
U1	3	200	1.000	42,9	114,3	166,7	107,9	85,9	111,2	114,1	103,7
U2	5	150	800	71,4	85,7	133,3	96,8	92,9	88,8	107,1	96,3
U3	7	175	600	100,0	100,0	100,0	100,0	100,0	100,0	100,0	100,0
U4	9	150	400	128,6	85,7	66,7	93,7	107,1	88,8	92,9	96,3
U5	11	200	200	157,1	114,3	33,3	101,6	114,1	111,2	85,9	103,7
Media	**7**	**175**	**600**	**100**	**100**	**100**		**100**	**100**	**100**	
S.q.m.	**2,8**	**22,4**	**282,8**	**40,4**	**12,8**	**47,1**		**10**	**10**	**10**	

In the processing into index numbers indicator X3 'weighs' more, due to the greater variability

Aggregation functions

Mazziotta-Pareto Index (MPI)

Penalization

The aggregation function (arithmetic mean of standardized values) is 'corrected' by a penalty coefficient that depends, for each unit, on the variability of the indicators compared to the average value ('horizontal variability')

This variability, measured by the coefficient of variation (CV), allows to penalize the score of the units that, with the same arithmetic average, have a higher imbalance between the values of the indicators.

The penalty can be added or subtracted, depending on the type of the studied phenomenon (poverty, development, etc.).

Aggregation functions

Mazziotta-Pareto Index (MPI)

The steps to compute the composite index are:

1) Given the original matrix **X**={x_{ij}}, the matrix of z-score **Z**={z_{ij}} is composed , where:

$$z_{ij} = \frac{(x_{ij} - M_{x_j})}{S_{x_j}} S + M$$

and

$$M_{x_j} = \frac{\sum_{i=1}^{n} x_{ij}}{n} \qquad S_{x_j} = \sqrt{\frac{\sum_{i=1}^{n} (x_{ij} - M_{x_j})^2}{n}}$$

x_{ij} is the value of the indicator *j* and the unit *i*;
S=10 e M =100

Aggregation functions

Mazziotta-Pareto Index (MPI)

2) Given the matrix **Z**={z_{ij}}, the vector **CV**={cv_i} is computed:

$$cv_i = \frac{S_{z_i}}{M_{z_i}}$$

where

$$M_{z_i} = \frac{\sum_{j=1}^{m} z_{ij}}{m} \qquad S_{z_i} = \sqrt{\frac{\sum_{j=1}^{m} (z_{ij} - M_{z_i})^2}{m}}$$

3) The composite index is:

$$MPI_i^{+/-} = M_{z_i}\left(1 \pm cv_i^{\,2}\right) = M_{z_i} \pm S_{z_i} cv_i$$

where the sign ± depends on the considered phenomenon.

Aggregation functions

Mazziotta-Pareto Index

Pros and Cons

This method assigns implicitly to all indicators the same weight, eliminating the variability.

It is applicable to any phenomenon, changing the sign of the penalty.

The index can be decomposed into two parts: the average effect (compensatory) and the penalty effect (imbalance).

It is possible to make only relative comparisons of the values of the units, respect to the average, over time.

Example

MPI (positive and negative penalty)

Unità	Indicatori			Var. standardizzate			Media	S.q.m.	CV	MPI+	MPI-
	X1	X2	X3	Z1	Z2	Z3					
U1	3	1	10	85,9	84,2	114,1	94,7	13,7	0,145	96,7	92,7
U2	5	3	8	92,9	100,0	107,1	100,0	5,8	0,058	100,3	99,7
U3	7	3	6	100,0	100,0	100,0	100,0	0,0	0,000	100,0	100,0
U4	9	3	4	107,1	100,0	92,9	100,0	5,8	0,058	100,3	99,7
U5	11	5	2	114,1	115,8	85,9	105,3	13,7	0,131	107,1	103,5
Media	**7**	**3**	**6**	**100**	**100**	**100**					
S.q.m.	**2,8**	**1,3**	**2,8**	**10**	**10**	**10**					

For U2, we have: $MPI_2^+ = 100 + 5{,}8(0{,}058) = 100{,}3$

and: $MPI_2^- = 100 - 5{,}8(0{,}058) = 99{,}7$

Aggregation functions

Adjusted MPI (AMPI)

It is variant of MPI, based on a transformation Min-Max instead in waste z-score

The transformation Min-Max is based on two goalposts: a minimum and a maximum that represent the possible range of variation of each indicator throughout the period considered and all units

AMPI vs MPI

MPI → the value 100 represents the mean of the values of all units
AMPI → the value 100 represents the mean of the *goalposts*.
You can 'fix' the goalposts in order to put equal to a 100? reference value (eg., the national average in a given year).

MPI → the normalized indicators have equal variances
AMPI → the normalized indicators have similar variances

Example

Comparison between normalization methods

Unità	Indicatori		*Z*-scores		Min-Max 0-1		Numeri indice (base=100)	
	X1	X2	Z1	Z2	R1	R2	I1	I2
A	3,0	10,0	-1,4	0,4	0,0	0,7	42,9	102,0
B	5,0	9,5	-0,7	-0,6	0,3	0,3	71,4	96,9
C	7,0	10,0	0,0	0,4	0,5	0,7	100,0	102,0
D	9,0	9,0	0,7	-1,6	0,8	0,0	128,6	91,8
E	11,0	10,5	1,4	1,4	1,0	1,0	157,1	107,1
Media	**7,0**	**9,8**	**0,0**	**0,0**	**0,5**	**0,5**	**100,0**	**100,0**
S.q.m.	**2,8**	**0,5**	**1,0**	**1,0**	**0,4**	**0,3**	**40,4**	**5,2**
C.V. (%)	**40,4**	**5,2**	**-**	**-**	**70,7**	**63,7**	**40,4**	**5,2**

Tranformation in *z*-score (es., MPI) = equal variances
Trasformazione Min-Max (es., AMPI) = similar variances

Aggregation functions

Adjusted MPI (AMPI)

Normalization

Given the original matrix **X**={x_{ij}}, the normalized matrix R={r_{ij}} is formed, by the following formula:

$$r_{ij} = \frac{(x_{ij} - Min_{x_j})}{(Max_{x_j} - Min_{x_j})} 60 + 70 \qquad (1)$$

where:
x_{ij} is the value of the indicator *j* and the unit *i*,
Min_{x_j} e Max_{x_j} are the *goalposts* of the indicator *j*.

If the indicator *j* has negative polarity, it is necessary to make the complement to 200 of the formula (1).

If Min_{x_j} and Max_{x_j} are the minimum and the maximum of all values of the indicators, the values r_{ij} will be included in the range 70-130.

Aggregation functions

Adjusted MPI (AMPI)

Calculation of the goalposts with a reference value

Inf_{x_j} = lower value of the indicator *j* throughout the considered period;
Sup_{x_j} = higher value of the indicator *j* throughout the considered period;
Rif_{x_j} = reference value of the indicator *j* (eg., the average of a given year).

The *goalposts* are:

$$\begin{cases} Min_{x_j} = Rif_{x_j} - \Delta_{x_j} \\ Max_{x_j} = Rif_{x_j} + \Delta_{x_j} \end{cases}$$

where:

$$\begin{cases} \Delta_{1x_j} = Sup_{x_j} - Rif_{x_j} \\ \Delta_{2x_j} = Rif_{x_j} - Inf_{x_j} \\ \Delta_{x_j} = (\Delta_{1x_j} + \Delta_{2x_j}) / 2 \end{cases}$$

In this case, the values r_{ij} will be included, roughly, in the range 70-130

Aggregation functions

Adjusted MPI (AMPI)

Aggregation

The composite index of the unit i is obtained by the following formula:

$$AMPI_i^{+/-} = M_{r_i} \pm S_{r_i} cv_i$$

where:

$$cv_i = \frac{S_{r_i}}{M_{r_i}} \; ; \qquad M_{r_i} = \frac{\sum_{j=1}^{m} r_{ij}}{m} \; ; \qquad S_{r_i} = \sqrt{\frac{\sum_{j=1}^{m} (r_{ij} - M_{r_i})^2}{m}} \; .$$

therefore, also the AMPI, is composed by two parts:

- Mean effect (M_{r_i})
- Penalty effect ($S_{r_i} cv_i$)

Aggregation functions

Adjusted MPI (AMPI)

Pros and cons

It is applicable to any phenomenon, changing the sign of the penalty.

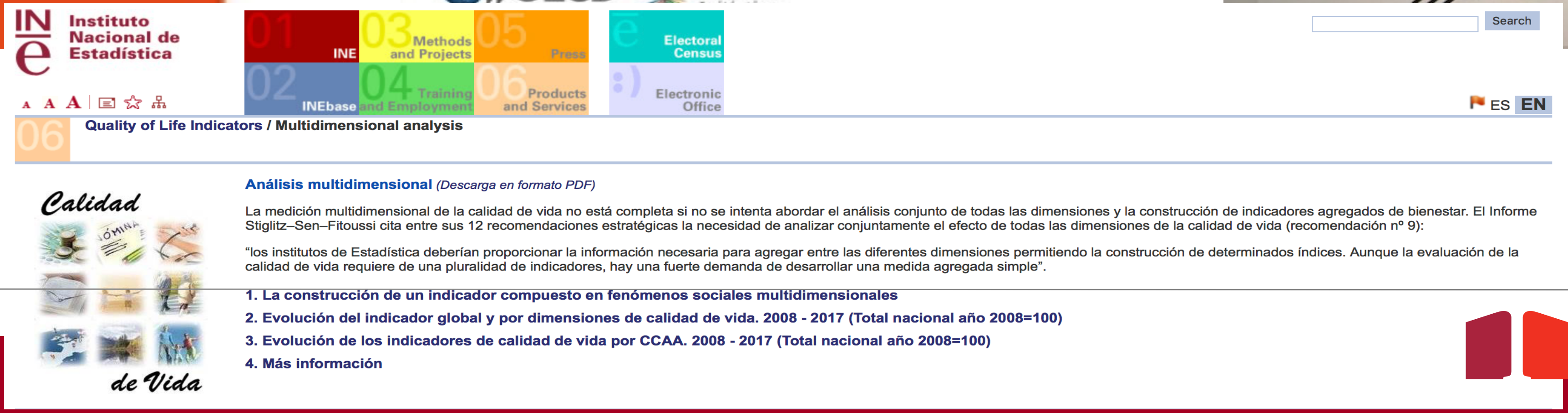
The index can be decomposed into two parts: the mean effect (compensatory) and the penalty effect (imbalance).

In order to calculate the composite index of a statistical unit, it is not necessary to know the values of the other.

It is possible to make absolute comparisons between units over time.

It does not completely purifies indicators from the variability effect.

Well-being in International Scenario



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06 **Quality of Life Indicators / Multidimensional analysis**

Calidad de Vida

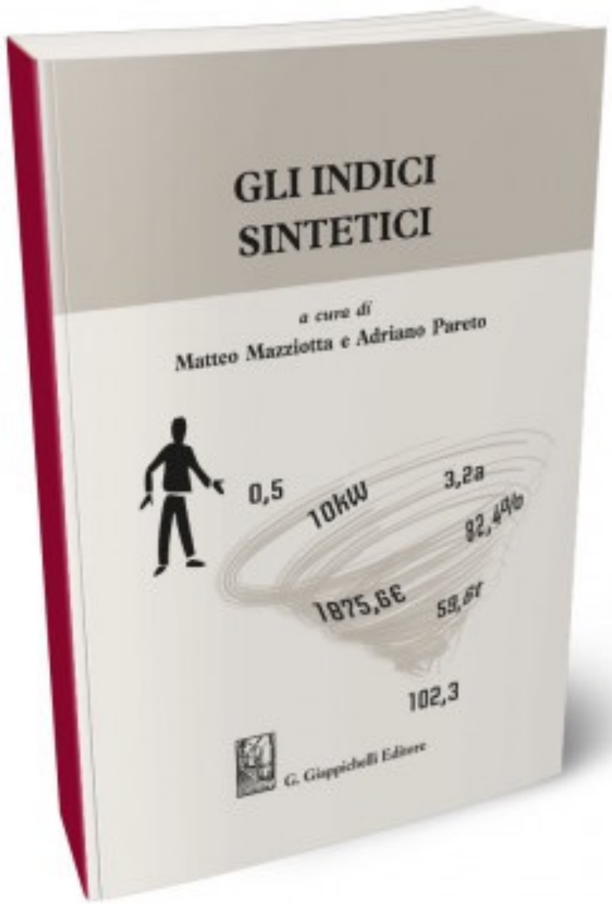
Análisis multidimensional *(Descarga en formato PDF)*

La medición multidimensional de la calidad de vida no está completa si no se intenta abordar el análisis conjunto de todas las dimensiones y la construcción de indicadores agregados de bienestar. El Informe Stiglitz–Sen–Fitoussi cita entre sus 12 recomendaciones estratégicas la necesidad de analizar conjuntamente el efecto de todas las dimensiones de la calidad de vida (recomendación nº 9):

"los institutos de Estadística deberían proporcionar la información necesaria para agregar entre las diferentes dimensiones permitiendo la construcción de determinados índices. Aunque la evaluación de la calidad de vida requiere de una pluralidad de indicadores, hay una fuerte demanda de desarrollar una medida agregada simple".

1. **La construcción de un indicador compuesto en fenómenos sociales multidimensionales**
2. **Evolución del indicador global y por dimensiones de calidad de vida. 2008 - 2017 (Total nacional año 2008=100)**
3. **Evolución de los indicadores de calidad de vida por CCAA. 2008 - 2017 (Total nacional año 2008=100)**
4. **Más información**

Conclusions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