











collapse, forming two clusters:  $\{2, 4, 5\}$ , associated with the  $\mu = 0$  borderline, and  $\{8, \dots, 14\}$ , associated with the  $\phi = 0$  borderline. In fact, as Critchley & Marriott (2016) noticed, problems are met near the frontier  $(\Theta - \hat{\Theta})$ . Incidentally, a similar phenomenon happened in large neighborhoods of the “MED” distribution (see Figure 1).

## 6. Conclusions

We have shown that:

- the critical locus of level  $1 - \alpha$  associated with the GOF test  $\mathfrak{L}^{\hat{\theta}} \stackrel{?}{=} \mathfrak{L}^{\theta^0}$  is an ellipsoid  $\mathcal{E}_{\theta^0}(\alpha) \subset \Theta$  which depends (centre, eccentricity) on  $\theta^0$
- the image of  $\mathcal{E}_{\theta^0}(\alpha)$  under  $\mathfrak{P} : \Theta \rightarrow NB(D_{\mathcal{R}})$  associating to  $\theta$  the probability  $\mathfrak{L}^{\theta}$  is theoretically a metric sphere
- $\mathfrak{P}(\mathcal{E}_{\theta^0}(\alpha))$  can be considered as spherical only when  $\mathcal{E}_{\theta^0}(\alpha)$  is “far enough” from the frontier of  $\Theta$  (see Figure 1, or Critchley & Marriott (2016)).

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