OPTIMAL ECONOMIC AGE REPLACEMENT MODELS FOR NON-REPAIRABLE SYSTEMS WITH SUDDEN BUT NON-CONSTANT FAILURE RATE

A Presentation

by

Dr Nse Udoh
University of Uyo, Nigeria

PRESENTATION OUTLINES

- **PART ONE: INTRODUCTION AND REVIEW OF RELATED LITERATURE**
- BACKGROUND ON:
- MATHEMATICAL THEORY OF RELIABILITY
- MAINTENANCE THEORY OF RELIABILITY
- REPAIRABLE AND NON-REPAIRABLE SYSTEMS
- REVIEW OF RELATED LITERATURE
- OBJECTIVES OF THE STUDY
- **PART TWO: METHODOLOGY**
- **PART THREE: APPLICATION AND RESULTS**
- **PART FOUR: DISCUSSION AND CONCLUSION**

Introduction

Mathematical Theory of Reliability

- Mathematically, reliability theory is a body of ideas, mathematical models, and methods directed toward the solution of problems in predicting, estimating, or optimizing the probability of survival, mean life, or more generally, life distribution of components or systems.
- Other problems considered in reliability theory are those involving the probability of proper functioning of the system at either specified or an arbitrary time, Barlow and Proschan (1965).

Maintenance Theory of Reliability

- Maintenance theory of reliability is primarily concerned with the formulation of either schedule (preventive) maintenance schemes or off-schedule (corrective) maintenance schemes and sometimes both depending on the type and nature of the equipment.
- The schedule or preventive maintenance (PM) is done at regular intervals – fixed or sequential.
- Its main aim is to ensure that repairable systems work within specified level of performance, reliability and safety.
- This mission is achieved by inspections, and minor or major overhaul in order to prevent systems failure rates from increasing above design level.
- Whereas, off-schedule or corrective maintenance (CM) is necessitated by system in-service failure or malfunction. Its purpose is to restore system operation as soon as possible by replacing, repairing or adjusting the component(s) which cause(s) failure – interruption of service.

Introduction Condt.

System Failures and Failure Distributions

- Causes of failures in many complex systems may be probabilistic, and as such, probability theory provides the underlying approach to solving its associated maintenance problem.
- However, if interest is on the dynamic behaviour of the system failures as a function of time, then its stochastic processes in terms of failure and repair are considered, Pharm (2003).
- Various distributions are exhibited by different systems based on their failure mode.
- For instance, systems with constant failure rate is best modeled by the exponential distribution. E.g. light bulb, etc.
- Also, we have systems which undergoes sudden and complete but non constant failure rate.
- They are modeled using any of lognormal distribution, Log-Logistic distribution, Birnbaum-Saunders distribution, among others. Examples of such failures are seen in electronic systems.
- We also have those systems whose component's operational efficiency deteriorates with time and usage; this kind of failure is modeled using the gamma distribution, Weibull distribution, among others.

Types of Maintenance

Maintenance actions are classified according to their associated impact on the operating conditions of the systems; namely:

- Perfect maintenance otherwise known as "as-good-as-new" e.g. complete overhaul of engine.
- Minimal maintenance otherwise known as "same-as-old" e.g. changing a flat tyre and fan belt of a vehicle
- Imperfect (normal maintenance) – "better-than-old" e.g. engine tune-up
- Worse maintenance – causes the system to perform at a level "worse-than-old" but not capable of total breakdown, e.g. any faulty action in the cause of imperfect maintenance.

Introduction Condt.

- Worst maintenance – which causes the system to fail or breakdown, e.g. repair of wrong parts due to wrong diagnosis and partial repair of faulty parts, Udoh (2016).

Repairable and Nonrepairable Systems

- **Repairable Systems**: These are systems that can be restored to their original or acceptable operational condition after failure or degradation.
- These operations may include servicing, adjustment of settings or the replacement of parts or components,, etc.

- Examples of such systems include automobiles, industrial machinery, and other mechanical equipment such as power generators.

- **Non-repairable systems:**

- These are systems whose maintenance involve the complete replacement of components or the entire system.
- These systems are mainly electronic and include stabilizers, photocopy machines, transmitters, among others.
- Non-repairable systems are therefore characterized by specific failure time distributions and their derived probability functions, (Udoh et al., 2022).

Objectives of the study

- To propose new preventive replacement maintenance models for nonrepairable systems

- To determine the required time for preventive replacement maintenance such that the total expected replacement cost per unit time, $E[C(\tau)]$ is minimum.

- To apply both the existing and proposed models to a nonrepairable system with sudden but nonconstant failure rate (radio transmitter system).

- To Obtain an optimal preventive replacement policy for the radio transmitter system.

Brief Review of Related Literature

- According to Jardine and Buzacott (1985) [1] and Dekker (1996) [2], maintenance can be defined as "the combination of all technical and associated administrative actions intended to retain an item or system, or restore it to a state in which it can perform its required function."
- Maintenance does not only improve the cost-efficiency of operating a system but it can also significantly reduce the probability of catastrophic failure of the system, Marais and Saleh (2009) [3].
- Therefore, maintenance managers must plan the maintenance actions, so that a balance is achieved between the expected benefits and corresponding expected potential consequences, Zio (2009) [4].
- On the contrary, poor maintenance of production facilities can result in defective end-product and customer dissatisfaction, lost production runs, cost inefficiencies, and sometimes, unavailability of the facility for future use, Lavy et.al (2010) [5].
- It was further asserted by Bagshaw and George (2015) [6] that "facility maintenance is the effort in connection with different technical and administrative action to keep a physical asset, or restore it to a condition where it can perform a require function".
- Maintenance can be classified according to its type and its degree either as corrective maintenance (CM) and preventive maintenance (PM).
- For instance, in an age replacement maintenance policy, a unit is replaced at failure (CM) or at PM time, T where T is a constant, Wang and Pham (2006) [7] and Tam et. al (2007) [8].
- The principle of age replacement model was applied by Lai and Chen (2006) [9] in developing a periodic replacement policy for a two-unit system with failure rate interaction between units.
- E.T.C.

Methodology
Age-Based Preventive Replacement Model

- DECISION CRITERIA 1 - Expected Cost Rate
- $E[N(\tau)] = \frac{total\ expected\ cost}{replacement\ time}$
- DECISION CRITERIA 2 – Limiting Availability of the system
- Availability is the probability that a system will work as required during a particular period of time.
- $A(\tau^*) = \frac{E(UP)}{E(Up)+E(Down)}$

where $E[Up] = \int_0^\infty tf(t)dt + \tau^* R(\tau^*)$

$$\int_0^\infty tf(t)dt = \tau^* f(\tau^*) - 0 \times f(0) = \tau^* \times f(\tau^*)$$

$$E[Down] = D_{pr}F(\tau^*) + D_{fr}R(\tau^*)$$

Methodology Contd.

$D_{fr} = \frac{\sum_{i=1}^{n} D_i}{n}$ and $D_{pr} = \frac{\sum_{j=1}^{m} D_i}{m}$

Models formulation

(1) Failure-based Replacement Model: Bahrami et. Al (2000) [16] showed that the expected number of failures occurring per cycle time, (0, t) is equal to the probability of occurrence of failures before time, t denoted by F(t).

Let N(t) = discrete no. of failures occurring in the period, (0,t).

$$P[N(t) = n] = G(n); n = 0,1,2, \ldots$$

$$E[N(t)] = \sum_{N(t)=0}^{N(t)=n} N(t) \times G[N(t)]$$

Where $G[N(t)]$ is the failure distribution function of N(t) occurring in the period (0, t). It is assumed that each interval is made as short as the need may be so that the probability of having more than one failure is negligible. In this situation, the probability of having two failures is small compared to having a single failure and so on. That is;

$$P[N(t) = 1] > P[N(t) = 2] > P[N(t) = 3] > \ldots$$

$E[N(\tau)]$ is estimated thus;

$$G(1) = P[N(t) = 1] \approx F(\tau) and G(0) = P[N(t) = 0 \approx 1 - F(\tau)]$$

Then, $E[N(\tau)] = \sum_{n=0}^{\infty} n \times G(n) = 0 \times [1 - F(\tau)] + 1 \times F(\tau)$

$$E[N(\tau)] = F(\tau) \tag{1}$$

Eqn (1) is the cumulative probability of occurrence of failure before time, t.

Methodology Condt.

Let C_{fr} = the total cost of failure replacement,

C_{pr} = the total cost of preventive replacement maintenance,

τ = the replacement time.

The total expected cost per cycle for preventive replacement maintenance at replacement time, τ is defined as;

$$E[C(\tau)] = \frac{total\ expected\ cost}{replacement\ time} = \frac{C_{pr}+C_{fr}\{N(\tau)]}{\tau} \qquad (2)$$

Therefore;

$$E[C(\tau)] = \frac{C_{pr}+C_{fr}F(\tau)]}{\tau} \qquad (3)$$

2. Hazard-based Replacement Model: Cassady et. al (2003) [17] stated that the mean number of failures occurring during the cycle $(0,\tau]$ is equal to the cumulative hazard rate at time, τ using the *concept of non-homogeneous Poisson process.*

Hence,

$$E[C(\tau)] = \frac{C_{pr}+C_{fr}H(\tau)]}{\tau} \qquad (4)$$

Where $H(\tau)$ is the cumulative hazard function.

The propose age replacement models

Let C_r = the replacement maintenance costs for each failed unit (RM),

Methodology Condt.

C_p = the preventive maintenance cost for each non-failed unit, where $C_p < C_r$

$N_1(t)$ denote the number of failures in the interval (0, t],

$N_2(t)$ denote the number of non-failed units that are preventively maintained in the interval (0, t].

Therefore, the expected cost during (0, t] is expressed as;

$$E[C(\tau)] = C_r\ E\{N_1(t)\} + C_p E\{N_2(t)\}$$

Based on the renewal reward theorem, Ross (1970) [18], the expected cost per unit time (expected cost rate) for an infinite time span is;

$$E[C(\tau)] \equiv \lim_{T\to\infty} \frac{\hat{C}(T)}{t} = \frac{Expected\ cost\ of\ one\ cycle}{mean\ time\ of\ one\ cycle}$$

Let **T**, $(0 < T \leq \infty)$ be the time for a planned replacement of a component with failure time, τ which is the mean time of a cycle (where a cycle refers to the interval from the start of the system to the completion of repair maintenance action or replacement maintenance)

The expected cost on a cycle as obtained in Udoh et. al (2020) [19] is expressed as;

$$C_r P(\tau \leq T) + C_p P(\tau > T) = C_r F(T) + C_p F^{/}(T) \qquad \text{where,} \qquad F^{/}(T) = 1 - F(T)$$

Hence, our proposed expected cost function is given in (5).

Methodology Condt.

$$E[C(\tau)] = \frac{C_r F(T) + C_p F^{/}(T)}{\tau} \qquad (5)$$

- Similarly, if we also assume that the mean number of failures occurring during the cycle (0, τ] is equal to the cumulative hazard rate at time, τ using the concept of *non-homogeneous Poisson process* as in [17];
- The expected cost per unit time for preventive replacement maintenance is given by;

$$E[C(\tau)] = \frac{C_r F(T) + C_p H^{/}(T)}{\tau} \qquad (6)$$

An optimal policy can be found by obtaining the value of τ that minimizes these cost functions.

- **Minimization of the expected cost functions**

Taking the partial derive of (2) with respect to τ yields;

$$\tau^* = \frac{\frac{C_{pr}}{C_{fr}} + E[N(\tau)]}{\frac{d}{d\tau}E[N(\tau)]}$$

To obtain $\boldsymbol{\tau}^*$for the models in Eqns (3) and (4), we respectively have;

$$E[N(\tau)] = \frac{d}{d\tau}E[N(\tau)] = \frac{d}{d\tau}F(\tau) = f(t)$$

$$\tau^* = \frac{\frac{C_{pr}}{C_{fr}} + F(\tau)}{f(t)} \qquad (8)$$

Methodology Condt.

$$E[N(\tau)] = H(t);\ \frac{d}{d\tau}E[N(\tau)] = \frac{d}{d\tau}H(\tau) = h(t)$$

$$\therefore\ \tau^{*} = \frac{\frac{C_{pr}}{C_{fr}} + H(\tau)}{h(t)} \qquad (9)$$

Similarly, we also obtain $\boldsymbol{\tau}^{*}$ respectively from (5) and (6), as;

$$\tau^{*} = \frac{F(\tau)}{f(t)} + \frac{c_p}{(c_r - c_p)f(T)} \qquad (10)$$

$$\tau^{*} = \frac{H(\tau)}{h(t)} + \frac{c_p}{(c_r - c_p)h(T)} \qquad (11)$$

❖ Limiting Availability of a System

Availability is the probability that a system will work as required during a particular period of time.

- ❖ Let $A(\tau^{*})$ denote the availability of a system at optimal time, $\boldsymbol{\tau}^{*}$
- ❖ E[Up] denotes the expected uptime at optimal time, τ^{*}
- ❖ E[Down] denotes the expected downtime at optimal time, τ^{*}
- ❖ D_{pr} is the average downtime for preventive replacement
- ❖ D_{fr} is the average downtime for failure replacement

Methodology Condt.

- R(τ*) is the reliability at optimal time, τ^*
- F(τ*) is the cumulative failure at optimal time, τ^*

According to Cassady et. Al (2023) [17];

$$E[Up] = \int_0^\infty tf(t)dt + \tau^* R(\tau^*)$$

where

$$\int_0^\infty tf(t)dt = \tau^* f(\tau^*) - 0 \times f(0) = \tau^* \times f(\tau^*)$$

$$E[Down] = D_{pr}F(\tau^*) + D_{fr}R(\tau^*)$$

where

$$D_{fr} = \frac{\sum_{i=1}^{n} D_i}{n} \text{ and } D_{pr} = \frac{\sum_{j=1}^{m} D_i}{m}$$

$$A(\tau^*) = \frac{E(UP)}{E(Up)+E(Down)} = \frac{\tau^* f(\tau^*)+\tau^* R(\tau^*)}{\tau^* f(\tau^*)+\tau^* R(\tau^*)+\left(D_{pr}F(\tau^*)+D_{fr}R(\tau^*)\right)} \quad (12)$$

Application of Replacement Models to the Maintenance of Radio Transmitter System

- ❖ The radio transmitter is a complex electronic device which major components are integrated circuits, diodes and fuses which are replaced after each failure. Hence, it is a non-repairable system. It fails suddenly but at a non-constant rate.

❖ The Need for Failure Distribution

1) To provide parameters estimate

2) To obtain optimal probabilities for inter-failure times of the Transmitter System.

3) The estimates would be used to obtain optimal replacement policies for existing age preventive replacement maintenance models as well as the proposed models in this work.

❖ Choice of Failure Distribution of Equipment

Several parametric models have been successfully used as population model for failure times distribution of both repairable and nonrepairable systems associated with a wide range of products.

These distributions are exhibited by systems according to their mode of failure and the failure mechanism.

Therefore, choosing appropriate model for failure times distribution can either be based on probabilistic views of the physics of the failure mode or the success in fitting empirical data, Udoh et. al (2024).

Hence, the choice of Birnbaum-Sanders (Fatigue Life) as the failure distribution function that characterize the failure distribution of radio transmitter system in this work is based on the its empirical success.

Application

❖ Goodness-of –fit test

The inter-failure times of the transmitter system was modeled as the Birnbaum-Saunders distribution having a chi-square best fit of rank 1 using Easyfit (5.6) software.

❖ The Birnbaum-Saunders (Fatigue Life) Failure Distribution

The Birnbaum Sanders (BS) distribution has appeared in several different contexts, with varying derivations. It was given by Fletcher in 1911, and was formally obtained by Konstantinowsky (1914) [20].

- However, it was the derivation by Birnbaum and Sanders (1969a) [21] that brought the usefulness of this distribution into a clear focus.
- Furthermore, Birnbaum and Sanders(1969b) [22] introduced a two-parameter lifetime distribution to model fatigue life of a metal, subject to cyclic stress by making a monotone transformation on the standard normal random variable.
- Consequently, the distribution is also sometimes referred to as the *fatigue-life distribution*.
- Since then, extensive work has been done on this model providing different interpretations, constructions, generalizations, inferential methods, and extensions to bivariate and multivariate cases, Bhattacharyya and Fries (1982) [23].

❖ Failure and Cumulative Failure Distributions of the Two-Parameter Birnbaum-Saunders distribution

The probability density and cumulative distribution function of Birnbaum-Sanders distribution is respectively given as;

Application Condt.

$$f(t;\alpha,\beta)=\frac{1}{2\sqrt{2\pi\alpha\beta}}\left[\left(\frac{t}{\beta}\right)^{\frac{1}{2}}+\left(\frac{\beta}{t}\right)^{\frac{3}{2}}\right]exp\left[-\frac{1}{2\alpha^2}\left(\frac{t}{\beta}+\frac{\beta}{t}-2\right)\right];t>0;\alpha,\beta>0 \qquad (17)$$

$$F(t;\alpha,\beta)=\emptyset\left(\frac{1}{\alpha}\left[\sqrt{\frac{t}{\beta}}-\sqrt{\frac{\beta}{t}}\right]\right) \qquad (18)$$

❖ The Mean and Variance of the two-parameter Birnbaum-Saunders Distribution

In our work, we use the Monotone Transformation to obtain the mean and variance and the results are;

$$E[T]=\beta\left(1+\frac{2\alpha^2}{4}\right)$$

$$Var(T)=\beta^2\,\alpha^2\left(1+\frac{5\alpha^2}{4}\right)$$

❖ Estimation of parameters for the two-parameter Birnbaum-Saunders Distribution

The shape and scale parameters of the distribution, α and β were respectively obtained by the modified moment estimation method and the results are;

$$\hat{\beta}=\sqrt{Sr}=\sqrt{Sr}\left(1+\frac{\alpha^2}{2}\right)$$

and

$$\hat{\alpha}=\left\{2\left(\frac{s}{\sqrt{Sr}}-1\right)\right\}^{\frac{1}{2}}=\left\{2\left(\frac{s}{\hat{\beta}}-1\right)\right\}$$

Application Condt.

❖ Reliability Function of Birnbaum- Saunders Distribution

Reliability, R(t) is the probability that the item does not fail in the time interval (0,t], or, in other words, the probability that the item survives the time interval (0, t] and is still functioning at time t. It is given as;

$$R(t) = 1 - F(t) = P(T > t); t > 0$$

$$= 1 - \int_0^t f(u)du = \int_t^\infty f(u)du$$

$$= 1 - F(t; \alpha, \beta)$$

$$\therefore R(t) = \emptyset\left\{\frac{1}{\alpha}\left(\sqrt{\frac{\beta}{t}} - \sqrt{\frac{t}{\beta}}\right)\right\} \qquad (35)$$

❖ Hazard and Cumulative Hazard Functions of Birnbaum- Saunders Distribution

The hazard function gives the failure rate of the system immediately after time x. It is a function of time and has a probabilistic interpretation. The hazard and cumulative hazard functions are respectively given as;

$$h(t) = \frac{\left(\sqrt{\frac{t}{\beta}} - \sqrt{\frac{\beta}{t}}\right)}{2\alpha t} \times \left(\frac{\emptyset(Z)}{\emptyset(-Z)}\right) \qquad (36)$$

Application Condt. And Results

$$H(t) = -lnR(t) \tag{38}$$

Results

1. Estimation of the Birnbaum-Saunders Parameter of the Radio Transmitter System

The modified moment estimators as given in (33) and (34) were used to obtained estimated shape parameter;

$$\hat{\alpha} = 0.95701$$
$$\hat{\beta} = 557.37$$

2. Probability functions and Replacement models of radio transmitter systems at respective optimum times

Optimal probability functions in Eqs (15) and (17), availability factor in Eq (12) and expected cost per cycle in Eqs (3) - (6) were obtained in Table 1 at the respective optimum values of the four replacement models under consideration.

Results Condt.

Table 1: Optimal probabilities, availability and expected cost functions for replacement models

Probability Function	Bahrami et. Al (2000) failure-based model	Cassady et. Al (2003) hazard-based model	Udoh et. al (2022) failure-based model 1	Udoh et. al (2022) hazard-based model 2
Optimal time	$\tau^* = 143$	$\tau^* = 137$	$\tau^* = 153$	$\tau^* = 149$
$f(\tau^*)$	0.00115	0.001081	0.001376	0.001145
$F(\tau^*)$	0.06254	0.0560	0.093197	0.069341
$h(\tau^*)$	0.0012	0.0015	0.0015174	0.001231
$R(\tau^*)$	0.93744	0.94407	0.906803	0.930659
$A(\tau^*)$	0.9829	0.98	0.9597	0.95903
$E[C(\tau^*)]$	388	392	208	166

Estimated parameters of the Birnbaum-Saunders distribution

- The estimates $\hat{\alpha} = 0.95701$ and $\hat{\beta} = 557.37$ are the respective shape and scale parameters of the Birnbaum-Saunders distribution.
- The shape of the failure density function and the hazard function are governed by α. Also, the failure density function is unimodal for all values of α.
- The scale parameter is also known as the median of the distribution.
- As the shape parameter, α increases, the hazard rate and the failure density function of the distribution becomes more skewed to the right.

Choice of Optimal Replacement Models

- The proposed replacement maintenance models by the authors yield improved results and are therefore considered the preferred economic optimal model for maintenance policy of the system due to the following reasons;

1) ***Improved optimal operational time estimate before replacement***:

- The Bahrami et. al (2000) failure function-based replacement model from column (2) of Table 1 yields an optimal replacement time of 143 hours versus the proposed optimal replacement time of 153 hours, based on Udoh et. al (2022) model-1 with same parameters in column 4.

Results Condt.

- Also, the hazard function-based replacement model of Cassady et. al (2003) in column 3 of Table 1 shows that the radio transmitter system has an optimal replacement time of 137 hours versus the proposed optimal replacement time of 149 hours from Udoh et. al (2022) model-2 with the parameters of the same kind in column 5.

2) ***Improved expected minimum cost value for replacement maintenance***:

- The failure function-based replacement model of Bahrami et. al (2000) in column (2) of Table 1 yields an expected minimum cost value of 388 naira versus our proposed expected minimum cost value of 208 naira from Udoh et. al (2022) model-1 based on the same parameters.
- Also, the hazard function-based replacement model Cassady et. al (2003) in column 3 of Table 1 shows that the radio transmitter system has an expected minimum cost value of 396 naira versus the proposed expected minimum cost value of 166 from Udoh et. al (2022) model-2 based on the same parameters.

3) ***Comparative chance of failure occurrence:***

- The failure function-based replacement model of Bahrami et. al (2000) in column (2) of Table 1 yields a 0.12% chance of failure occurrence versus a 0.15% chance of failure occurrence obtained from our proposed model of Udoh et. al (2022) model-1 of same kind.
- But, the hazard function-based replacement model of Cassady et. al (2003) in column 3 of Table 1 yields a 0.15% chance of failure occurrence versus a lesser percentage of 0.12% chance of failure occurrence obtained from our propose model of Udoh et. al (2022) model-2 of the same kind.

Conclusion

Reasons 1-3 provide the support points for the choice of our proposed hazard function-based model as the preferred model in particular and the proposed class of models in general for the study.

Consequently, our proposed preventive replacement maintenance models are the optimal economic models with respect to time and cost as vital economic factors in formulating replacement maintenance policies for the radio transmitter and similar systems.

References

[1] Jardine, A. K. S and Buzacott, J. A. (1985). Equipment reliability and maintenance. *European Journal of Operational Research*, 19: 285-296.

[2] Dekker, R. (1996). Applications of maintenance optimization models: a review and analysis. *Reliability Engineering and System Safety*, 51: 229-240.

[3] Marais, K. B. and Saleh J. H. (2009). Beyond its cost, the value of maintenance; an analytical framework for capturing its net present value, *Reliability Engineering and System Safety*, 94: 644-657.

[4] Zio, E. Maintainability: A key to Effective and Maintenance Management. John Wiley and Sons, 2009.

[5] Lavy S, Garcia J. A. and Dixit M. K. (2010). Establishment of KPIs for facilities performance measurement: Review of literature. *Facilities*, 28(9/10): 440-464.

[6] Bagshaw K.B., George T. P. (2015). Facility management and organizational effectiveness of manufacturing firms in Rivers State Nigeria. *European Journal of Business Management*, 7(26): 67-89.

[7] Wang, H. and Pham, H. Reliability and Optimal Maintenance. Springer, 2006

[16] Bahrami-G, K., Price, J.W.H. and Mathew J. (2000). The constant - interval replacement Model for preventive maintenance: A new perspective. *International Journal of Quality and Reliability Management*, 17(8): 822-838.

[17] Cassady, C. R., Pohl, E. A. and Murdock, W. P. (2003). Selective maintenance modeling for industrial systems, *Journal of Quality in Maintenance Engineering*, 7(2): 104-117.

[18] Ross, S. M. Introduction to Probability Models. Academic Press, 1970.

[19] Udoh, N., Effanga E. and Onwukwe, C. (2020) Complementary optimal age maintenance (COAM) policy for repairable systems. *International Journal of Reliability and Safety*, 14:1–13.

[20] Konstantinowsky, D. (1914). Elektrische Ladungen und Brown'sche Bewegung Sehr Kleiner Metallteilchen in Gasen. *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften*, 123: 1697- 1752.

[21] Birnbaum, Z. W., Saunders, S. C. (1969a). A new family of life distribution. *Journal of Applied Probability*, 6: 319-327.

[22] Birnbaum, Z. W., Saunders, S. C. (1969b). Estimation for a family of life distributions with applications to fatigue, *Journal of Applied Probability*, 6: 328.

[23] Bhattacharyya, G. K., Fries, A. (1982). Fatigue failure models - Birnbaum-Saunders versus inverse Gaussian. *IEEE Transactions on Reliability*, 31:439-440.

[24] Balakrishnan, N., Saulo, H., Bourguignon, M., Zhu, X. (2017). On moment-type estimators for a class of log-symmetric distributions. *Computational Statistics*, 32: 1339-1355.

[25] Udoh, N., Uko, I and Bassey K. (2024). Effect of load on sequential imperfect preventive maintenance and replacement schedule of mechanically repairable machines. Journal of Reliability and Statistical Studies, 16(2):337-356.

[26] Pham, H. (2003). Handbook on Reliability Engineering. Springer-Verlag London Ltd.

[27] Udoh, N. S. (2016). Optimal imperfect preventive maintenance and replacement models for aging repairable systems. PhD Thesis (unpublished).

Thank You