Data Science 1
Probability
Sample Spaces and Events

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Introduction

The Sample Space is the set or collection of all possible outcomes from an experiment. Denoted $\mathcal{S}$.

Any collection of outcomes from the sample space is called an *Event*. Denoted E. (Note that mathematically, $E \subseteq \mathcal{S}$.)

This seems more complicated than it really is.

Event Example

Example: A fisherman is interested in what type of fish will be the next to be caught.

$$\mathcal{S} = \{\text{Bass, Shad, Catfish, Salmon, Snapper}\}$$

Here are a few events:

$E_1 = \{Bass\}$
$E_2 = \{Snapper\}$
$E_3 = \{Bass, Shad, Snapper\}$

Notice that an event need not just have one outcome.

Operations on Sets/Events

Often we will need to combine events together or perform various operations on them.

- Complementation E^c is all of the outcomes in $\mathcal{S}$ not in E.
- Union of events E and F, denoted $E \cup F$ and is all the outcomes in E **or** F.
- Intersection of E and F, denoted $E \cap F$ and is all the outcomes common to both E **and** F.

Complement Example

Example: A fisherman is interested in what type of fish will be the next to be caught.

$$\mathcal{S} = \{\text{Bass, Shad, Catfish, Salmon, Snapper}\}$$

If $E = \{Bass, Shad, Snapper\}$ then the complement is:

$$E^c = \{Catfish, Salmon\}$$

Pretty simple.

Union Example

Example: A fisherman is interested in what type of fish will be the next to be caught.

$$\mathcal{S} = \{\text{Bass, Shad, Catfish, Salmon, Snapper}\}$$

If $E = \{Bass, Snapper\}$ and $F = \{Catfish, Snapper\}$ then the union is:

$$E \cup F = \{Bass, Catfish, Snapper\}$$

Notice that *Snapper* is not in the Union twice.

Intersection Example

Example: A fisherman is interested in what type of fish will be the next to be caught.

$$\mathcal{S} = \{\text{Bass, Shad, Catfish, Salmon, Snapper}\}$$

If $E = \{Bass, Snapper, Salmon\}$ and $F = \{Catfish, Snapper, Salmon\}$ then the intersection is:

$$E \cap F = \{Snapper, Salmon\}$$

It is simply what the two sets have in common.

Combination Example 1

Example: A fisherman is interested in what type of fish will be the next to be caught.

$$\mathcal{S} = \{\text{Bass, Shad, Catfish, Salmon, Snapper}\}$$

If $E = \{Bass, Snapper, Salmon\}$ and $F = \{Catfish, Snapper, Salmon\}$ then the union is:

$$\begin{aligned} E^c \cup F^c &= \{Shad, Catfish\} \cup \{Shad, Bass\} \\ &= \{Shad, Bass, Catfish\} \end{aligned} \tag{1}$$

As long as you pay attention to what you are doing this is pretty easy.

Combination Example 2

Example: A fisherman is interested in what type of fish will be the next to be caught.

$$\mathcal{S} = \{\text{Bass, Shad, Catfish, Salmon, Snapper}\}$$

If $E = \{Bass, Snapper, Salmon\}$ and $F = \{Catfish, Snapper, Salmon\}$ then the intersection is:

$$\begin{aligned} E^c \cap F^c &= \{Shad, Catfish\} \cap \{Shad, Bass\} \\ &= \{Shad\} \end{aligned} \tag{2}$$

As long as you pay attention to what you are doing this is pretty easy.

Special Relationships

There are a few relationships one should understand before going too much further.

- Union with the complement of itself.

$$E \cup E^c = \mathcal{S}$$

- If events E and F have no outcomes in common then,

$$E \cap F = \{\} = \emptyset$$

  here $\emptyset$ is called the *Empty Set* or *Null Set*.
- There is also the operations of *Set Minus* or Set Subtraction. If E and F are sets then $E \setminus F$ are all the outcomes in E that are not common to F. This can also be written as:

$$E \setminus F = E \cap F^c$$

Summary

Working with the Sample Space and Events are pretty simple.

- Complements are just everything not in E.
- Unions are a form a set addition, the resulting sets tend to be bigger.
- Intersections are everything the events have in common. Typically the resulting event is smaller.

Paying attention is the key to understanding and working with these operators.