Data Science 1
Probability
Properties of Probabilites

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Introduction

From the previous videos we understand what a Sample Space $\mathcal{S}$ is and what events are.

Now we want to know how to assign a probability to an event E, denoted $P(E)$.

One way is to use the Empirical approach which looks at the proportion of the time the event was observed.

$$\hat{P}(E) = \frac{n(E)}{n}$$

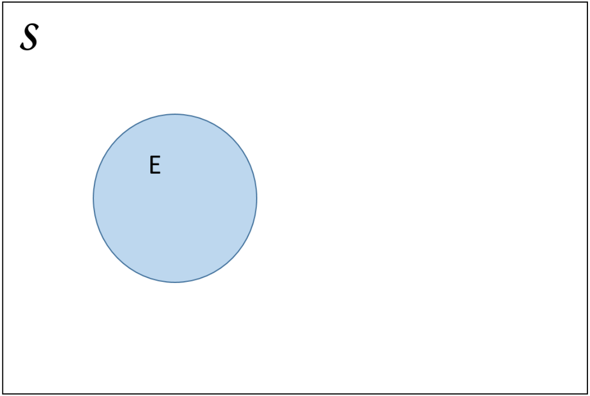
where n is the number of trials we observed and $n(E)$ is the number of those trials where E occured.

Here we use $\hat{P}$ to show that the probability is estimated from data.

However, this approach can be very time consuming and expensive as the experiment must be conducted and the outcomes must be observed and counted.

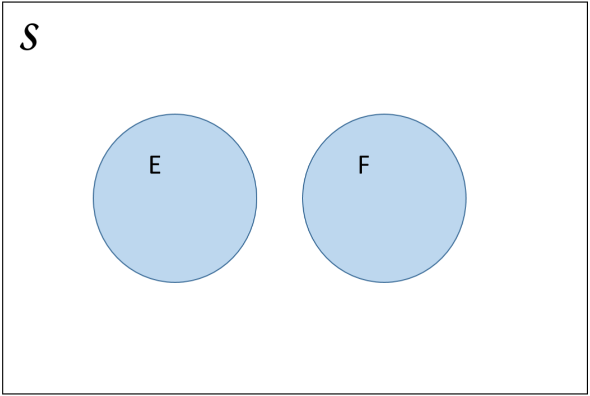
Venn Diagrams

Let E be an event in $\mathcal{S}$. Then we can represent this as a Venn Diagram.



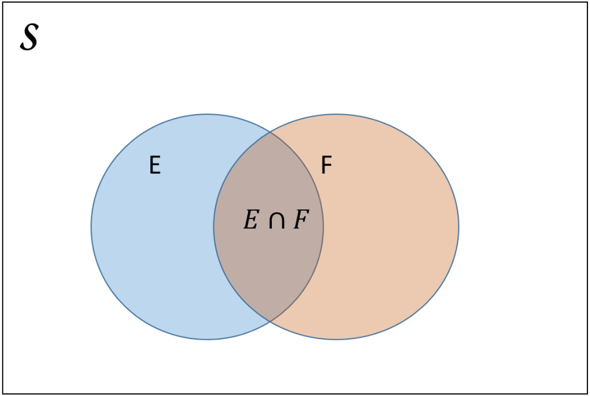
Venn Diagrams

Let E and F be events in $\mathcal{S}$. Then we can represent this as a Venn Diagram.



Venn Diagrams

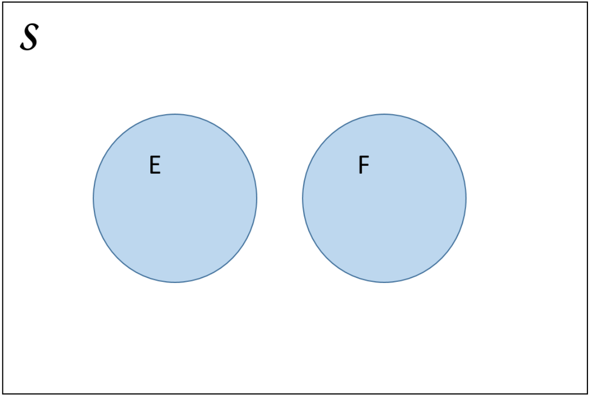
Let E and F be events in $\mathcal{S}$. The **intersection** of E and F is $E \cap F$



What the two events have in common.

Venn Diagrams

Let E and F be events in $\mathcal{S}$. Here the **intersection**is empty.



We would write this as: $E \cap F = \emptyset$

Venn Diagrams

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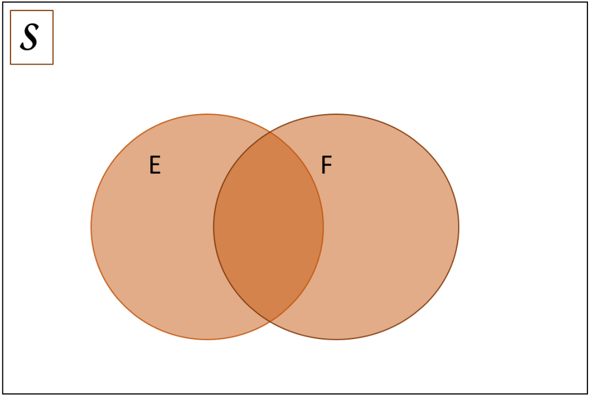
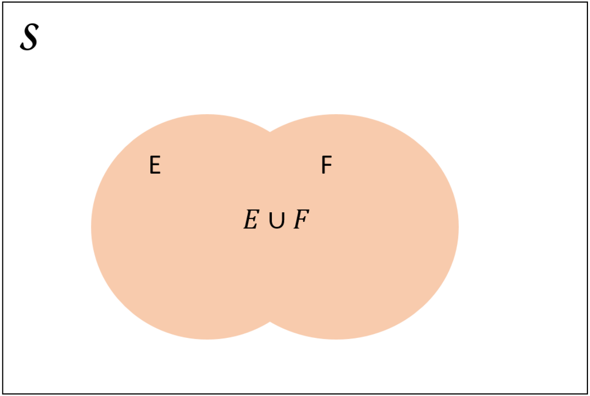


Suppose we want all events that are in E or in F?

Venn Diagrams

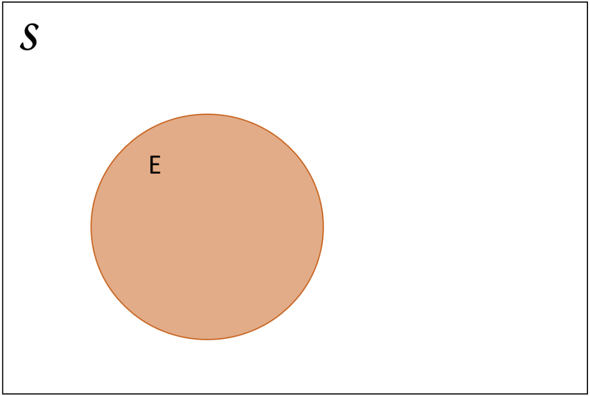
Let E and F be events in $\mathcal{S}$. The **Union** of E and F is $E \cup F$



The accumulation of all the outcomes in each event.

Venn Diagrams

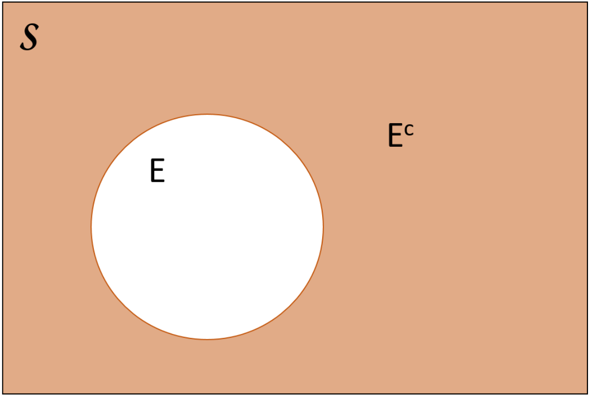
Let E be an event in $\mathcal{S}$.



What about all the other outcomes not in E?

Venn Diagrams

Let E an events in $\mathcal{S}$. The **complement** is all the objects not in E.



Axioms

Probability has three **Axioms** that hold it together:

1. $P(E) \geq 0$. All probabilities are non-negative.
2. $P(\mathcal{S}) = 1$. If I conduct the experiment one of the outcomes in $\mathcal{S}$ will happen.
3. If $E_1, E_2, E_3 ...$ are events in $\mathcal{S}$ such that $E_i \cap E_j = \emptyset$ for all $i \neq j$ then $P(E_1 \cup E_2 \cup E_3 \cup ...) = P(E_1) + P(E_2) + P(E_3) + ...$

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What does this mean?

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What does this mean?

Probability behaves like Area!

Summary

The knowing that Probability behaves like area gives us a way to think about the topic using ideas we already know.

- We need a few rules for probabilty that will help us out later.
- Drawing a Venn diagram can often help you solve problems.
- Think about probabilities as the area of the event in the Venn diagram with the total area of the Venn diagram being 1.