Data Science 1
Probability
Handy Probability Rules

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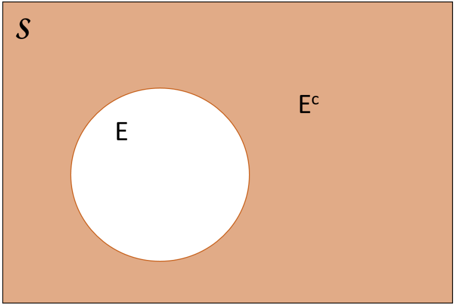
Introduction

Recall the following:

- E is an event in a sample space $\mathcal{S}$.
- $P(\mathcal{S}) = 1$.
- $E \cup F$ is the union of events E and F.
- $E \cap F$ is the intersection of events E and F.
- Probability behaves like area.

Complements

Suppose we know $P(E)$ and we want to know $P(E^c)$.



From here we can easily see that:

$$1 = P(\mathcal{S}) = P(E \cup E^c) = P(E) + P(E^c)$$

This implies:

$$P(E^c) = 1 - P(E)$$

Complements

Example: Suppose we know that the probability that my mobile phone has service along my path to work is 0.97. What is the probability that my mobile phone won't have service on my path to work?

Let E be the event that I have mobile service on my path to work. I know $P(E) = 0.97$.

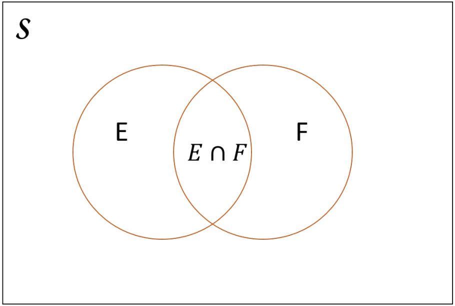
Here E^c is the event that I don't have mobile service on the path to work. We can calculate it by:

$$P(E^c) = 1 - P(E) = 1 - 0.97 = 0.03$$

Hence there is a probability of 0.03 that I don't have mobile service on the way to work.

Unions

Suppose we know $P(E)$, $P(F)$ and $P(E \cap F)$ and we want to know $P(E \cup F)$.



This gives us:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Unions

Football is a big deal at Sports R' Us sporting retailer. Corporate wide they have several groups that get together to watch Football. Of their employees we know that 12% support Manchester United, 19% support Chelsea and 3% support both Manchester United and Chelsea. What is the probability that a randomly selected employee supports Manchester United or Chelsea?

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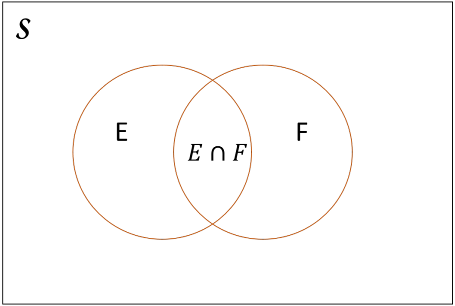
Using the rule we get:

$$P(M \cup C) = P(M) + P(C) - P(M \cap C) = 0.12 + 0.19 - 0.03 = 0.28$$

Intersections

Suppose we know $P(E)$, $P(F)$ and $P(E \cup F)$ and we want to know $P(E \cap F)$.



This gives us:

$$P(E \cap F) = P(E) + P(F) - P(E \cup F)$$

Intersections

Javier is interested in selling two hair care products, Brand A and Brand B. He knows that the probability that a person finds Brand A acceptable is 0.4 and the probability that a person finds Brand B acceptable is 0.6 and that the probability that find either Brand A or Brand B is acceptable is 0.72. What is the probability that a person finds both Brand A and Brand B acceptable?

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$$\begin{aligned} P(\text{Brand A}) &= 0.4 \\ P(\text{Brand B}) &= 0.6 \\ P(\text{Brand A} \cup \text{Brand B}) &= 0.45 \end{aligned}$$

We want $P(\text{Brand A} \cap \text{Brand B})$...

Intersections

What is the probability that a person finds both Brand A **and** Brand B acceptable?

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\end{aligned}
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We want $P(\text{Brand A} \cap \text{Brand B})$...

$$
\begin{aligned}
P(\text{Brand A} \cap \text{Brand B}) &= P(\text{Brand A}) + P(\text{Brand B}) - P(\text{Brand A} \cup \text{Brand B}) \\
&= 0.4 + 0.6 - 0.45 \\
&= 0.55
\end{aligned}
$$

Summary

Probabilities can be easily worked out given you know the correct information.

- $P(E^c) = 1 - P(E)$
- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- $P(E \cap F) = P(E) + P(F) - P(E \cup F)$

Depending on what you know you can find other probabilities.