Data Science 1
Probability
Counting Methods 1

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Introduction

From the last few videos it is clear that counting is very important.

Multiplication Rule

If we have two experiments E_1 and E_2 where E_1 can occur in n_1 ways and E_2 can occur in n_2 ways then the combined experiment can occur in $n_1 \times n_2$ ways.

Example

We have already seen this with the example with the dice. If we roll two dice it is a combined experiment of rolling one dice two times. Recall:

$$\mathcal{S} = \{⚀, ⚁, ⚂, ⚃, ⚄, ⚅\}.$$

Hence, are $n_1 = 6$ for roll one and $n_2 = 6$ for roll two which gives

$$n_1 \times n_2 = 6 \times 6 = 36$$

As before we have 36 possible outcomes.

This is easier to calculate that how we came up with them last time.

Example

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How many choices do I have for the first digit? $n_2 = 10$
How many choices do I have for the second letter? $n_3 = 26$

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How many choices do I have for the second letter? $n_3 = 26$
How many choices do I have for the third letter? $n_4 = 26$

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How many choices do I have for the third letter? $n_4 = 26$
How many choices do I have for the second digit? $n_5 = 10$

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How many choices do I have for the third letter? $n_4 = 26$
How many choices do I have for the second digit? $n_5 = 10$
How many choices do I have for the third digit? $n_6 = 10$

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How many choices do I have for the second digit? $n_5 = 10$
How many choices do I have for the third digit? $n_6 = 10$

Hence the combined experiment has $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6$ $= 26 \times 10 \times 26 \times 26 \times 10 \times 10 = 17,576,000$ possible outcomes.

Sampling with out replacement

In the above examples we were using a *Sampling with Replacement* approach. This means that once an outcome has been observed in the combined experiment it can be observed again.

In *Sampling with out replacement* once an outcome is seen it cannot be seen again.

Example

You and I go to a trendy new cafe for tea. The waiter brings a box of tea and upon inspection we see that there are seven tea bags all of which are different flavors. Hence if you have Earl Grey then I cannot and similarly if I have Mint then you cannot. We agree to let you choose first.

Example

You and I go to a trendy new cafe for tea. The waiter brings a box of tea and upon inspection we see that there are seven tea bags all of which are different flavors. Hence if you have Earl Grey then I cannot and similarly if I have Mint then you cannot. We agree to let you choose first.

How many choices of tea do you have? $n_1 = 7$

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You and I go to a trendy new cafe for tea. The waiter brings a box of tea and upon inspection we see that there are seven tea bags all of which are different flavors. Hence if you have Earl Grey then I cannot and similarly if I have Mint then you cannot. We agree to let you choose first.

How many choices of tea do you have? $n_1 = 7$
How many choices of tea do I have? $n_2 = 6$

Example

You and I go to a trendy new cafe for tea. The waiter brings a box of tea and upon inspection we see that there are seven tea bags all of which are different flavors. Hence if you have Earl Grey then I cannot and similarly if I have Mint then you cannot. We agree to let you choose first.

How many choices of tea do you have? $n_1 = 7$
How many choices of tea do I have? $n_2 = 6$

Hence there is $n_1 \times n_2 = 7 \times 6 = 42$ ways we could have tea.

Summary

The Multiplication rule allows us to count all the possible number of outcomes with out enumerating every single outcome.

Sampling **with** replacement allows for an outcome to reappear in the combined experiment.
Sampling **with out** replacement does not allow for an outcome to reappear in the combined experiment.