Data Science 1 Probability Counting Methods 2 Permutations

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Introduction

From the last few videos we have seen that being able to count outcomes without enumerating them is very important.

Multiplication Rule: If E_1 can occur n_1 ways and E_2 can occur n_2 ways then the combined experiment can occur in $n_1 \times n_2$ ways.

Sampling with Replacement: An outcome can appear more than once in the combined experiment.

Sampling with out Replacement: An outcome can only appear once in the combined experiment.

Permutations

When you think of the word *Permutation*, think *ordering*.

We are interested counting the number of ways we can permute (order) a set of objects.

This is naturally sampling without replacement since once an object is placed in an ordering it cannot be placed in a different position.

Factorial: Suppose we have n objects to order and by using the multiplication rule we have.

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$$

The ! is the *factorial* operator. Hence $n!$ reads as n *factorial*.

Example

At a ceremony there will be seven speeches given. How many possible orderings of the speeches are there?

Here $n = 7$

$$n! = 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$$

Hence there are 5,040 possible orderings of the speeches.

Example

Extend the previous example. Suppose you are a speaker at the ceremony. How many ways can you be the third speaker?

$$6 \times 5 \times 1 \times 4 \times 3 \times 2 \times 1 = 6! = 720$$

Hence there are 720 ways where you are the third speaker.

If we wanted to know the probability that you are the third speaker then we can consider E the event that you are the third speaker.

$$P(E) = \frac{|E|}{|\mathcal{S}|} = \frac{6!}{7!} = \frac{720}{5040} = \frac{1}{7}$$

Ordering a Subset

Suppose we don't want to order all the objects? For example, at the last minute the ceremony can only have 3 speakers. How many ways can the order the speeches of seven possible speakers into three spots?

$$7 \times 6 \times 5 = 210$$

In general if we have n objects that will be ordered into k positions we can use:

$$\begin{aligned} n(n-1)\cdots(n-k+1) &= \\ &= \frac{n(n-1)\cdots(n-k+1)(n-k)\cdots 3\cdot 2\cdot 1}{(n-k)(n-k-1)\cdots 3\cdot 2\cdot 1} \\ &= \frac{n!}{(n-k)!} \\ &= {}_nP_k \end{aligned}$$

Example

A bakery sells 14 different types of bread and in their window they have 4 bins to put bread on display. How many ways can they position the bread in the window?

$$ {}_nP_k = {}_{14}P_4 = \frac{14!}{(14-4)!} = \frac{14!}{10!} = 14 \cdot 13 \cdot 12 \cdot 11 = 24,024 $$

Hence there are 24,024 different possible displays in the window.

Summary

The Multiplication rule allows us to count all the possible number of outcomes with out enumerating every single outcome.

Permutations are *orderings* of the objects.

We can order a subset of the total number of elements using ${}_nP_k$.

Be careful using factorials. These numbers are **huge** and may cause calculation errors on your computer.