Data Science 1
Probability
Counting Methods 3
Combinations

Edward L. Boone

Introduction

From the last few videos we have seen that being able to count outcomes without enumerating them is very important.

Multiplication Rule: If E_1 can occur n_1 ways and E_2 can occur n_2 ways then the combined experiment can occur in $n_1 \times n_2$ ways.

Permutations: How many ways can I order n objects?

$$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$$

Permutation of Subset: How many ways can I order k objects from a collection of n objects:

$$_nP_k = \frac{n!}{(n-k)!} = n(n-1)(n-2)\cdots(n-k+1)$$

Combinations

When you think of the word *Combination*, think *pulling a subset*... **not** ordering. Suppose we have n objects that we wish to pull a sample of k objects from. How many ways can I do this?

This is naturally sampling without replacement since once an object is pulled it cannot be pulled again.

This is very similar to *permutations* however, order doesn't matter. Hence we can just divide by the number of ways we can order the k objects , which is $k!$. Hence we have:

$$ {}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!} $$

Example

I am planning on baking an apple pie which requires 3 apples each of different varieties. I go to the market and find that there are 12 varieties of apples. Since all the apples will be mixed together in the pie, the order doesn't matter. How many ways can I choose the three apples?.

Here $n = 12$ and $k = 3$ and hence:

$$\begin{aligned}
\binom{n}{k} &= \binom{12}{3} && (1)\\
&= \frac{12!}{3!(12-3)!} && (2)\\
&= \frac{479,001,600}{6(362,880)} && (3)\\
&= 220 && (4)
\end{aligned}$$

There are 220 ways I can choose the three varieties of apples.

Example

CodeDynamics has a team of 25 computer programmers on staff that are well versed in Python programming. A customer approaches the company with a project that you determine will require 5 programmers to complete the project. How many ways can you create this team of 5 programmers from the group of 25 programmers?

Here $n = 25$ and $k = 5$ and hence:

$$\begin{aligned} \binom{n}{k} &= \binom{25}{5} \\ &= \frac{25!}{5!(25-5)!} \\ &= \frac{1.55 \times 10^{25}}{120(2.43 \times 10^{18})} \\ &= 53,130 \end{aligned}$$

There are 53,130 possible teams of 5.

Summary

The Multiplication rule allows us to count all the possible number of outcomes with out enumerating every single outcome.

Permutations are *orderings* of the objects.

Combinations are *subsets* of the objects (order does not matter).

Be careful using factorials. These numbers are **huge** and may cause calculation errors on your computer.