Data Science 1
Probability
Bernoulli Experiments

Edward L. Boone

Introduction

Multiplication Rule: If E_1 can occur n_1 ways and E_2 can occur n_2 ways then the combined experiment can occur in $n_1 \times n_2$ ways.

Permutations: How many ways can I order n objects?

$$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$$

Permutation of Subset: How many ways can I order k objects from a collection of n objects:

$${}_nP_k = \frac{n!}{(n-k)!} = n(n-1)(n-2)\cdots(n-k+1)$$

Combinations How many ways can I pull k objects from n distinct objects:

$${}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Bernoulli Experiment

A *Bernoulli* experiment is one where there are exactly two outcomes in the sample space.

We have already seen $\mathcal{S} = \{\text{Goal}, \text{No Goal}\}$.

Here are a few more:

- $\mathcal{S} = \{\text{On}, \text{Off}\}$
- $\mathcal{S} = \{\text{Alive}, \text{Dead}\}$
- $\mathcal{S} = \{\text{Pregnant}, \text{Not Pregnant}\}$
- $\mathcal{S} = \{\text{Success}, \text{Failure}\}$

Notice that there are only two possible outcomes and there is no “in between” on any of the states.

Bernoulli Random Variable

Since there are only two outcomes in the sample space the Bernoulli random variable is typically:

$$X = \begin{cases} 1 & \text{if A is observed} \\ 0 & \text{otherwise} \end{cases}$$

This formulation will allow for easy calculations later.
Examples:
$\mathcal{S} = \{\text{On}, \text{Off}\}$

$$X = \begin{cases} 1 & \text{if On} \\ 0 & \text{otherwise} \end{cases}$$

Bernoulli Random Variable

Since there are only two outcomes in the sample space the Bernoulli random variable is typically:

$$X = \begin{cases} 1 & \text{if A is observed} \\ 0 & \text{otherwise} \end{cases}$$

This formulation will allow for easy calculations later.
Examples:
$\mathcal{S} = \{\text{On}, \text{Off}\}$

$$X = \begin{cases} 1 & \text{if On} \\ 0 & \text{otherwise} \end{cases}$$

$\mathcal{S} = \{\text{Success}, \text{Failure}\}$

$$X = \begin{cases} 1 & \text{if Success} \\ 0 & \text{otherwise} \end{cases}$$

Bernoulli Probability Distribution

If we want to turn Bernoulli experiments into to probabilities we can make use of the idea of a *Probability Distribution* ,f, which is a function that assigns probabilities to each value of the random variable.
$f(x) = P(X = x)$. This is easy for a Bernoulli Random Variable.

$$f(x) = P(X = x) \begin{cases} p & \mathrm{X} = 1 \\ 1 - p & \mathrm{X} = 0 \end{cases}$$

where here p is the probability of $X = 1$ which is the probability of Success, On, or what ever outcome has been assigned the value 1. Note that $f(0) = 1 - p$ since the we know that probabilities must sum to one.

$$f(1) + f(0) = 1 \Rightarrow f(0) = 1 - f(1) \Rightarrow f(0) = 1 - p$$

Summary

The Bernoulli experiment and its associated random variable and probability distribution are simple but extremely important in understanding probability.

- It is the simplest type of experiment with only two possible outcomes.
- The random variable is easy to define just using 0 and 1.
- The probability distribution is easy to understand and only requires knowledge of the probability of one of the two possible outcomes.

We didn't need any counting schemes in this...