Data Science 1
Probability
Poisson Experiments

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Introduction

Items to remember:

Permutations: Number of ways you can arrange k objects is
$k! = k \times (k-1) \times ... \times 3 \times 2 \times 1$.

Probability Distribution: How are probabilities assigned to the values of the random variable? $f(x) = P(X = x)$.

Bernoulli Experiment: Two outcomes where exactly one or the other occurs, typically denoted as Success or Failure. $\mathcal{S} = \{\text{Success}, \text{Failure}\}$

Poisson Experiment

A *Poisson* experiment is one where we count the number of Bernoulli outcomes that occur over a specified amount of time or space.

- In the previous experiments the experimenter intervenes to illict a Bernoulli response.
- The process that generates the Bernoulli outcomes happens without our intervention.
- The amount of time is specified by us.

Poisson Examples

Here are some examples of Poisson experiments:

- The number of customers who come to a sales stall per day.
- The number of fish caught per day.
- The number of cars that drive through an intersection per hour.
- The number of website hits per minutes.
- The number of defects in fabric per metre.

Notice that we are counting so the random variable X assign a whole number.

Notice that time or space is a continuous measurement.

Poisson Probability Distribution

The Poisson probability distribution is given by:

$$f(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, 4, ...$$

where λ is the *m*ean number of sucesses observed over the specified time.

- Notice that x is defined over whole numbers and zero is included. (we need to account for the fact that the event might not occur in the time interval.)
- Here λ must be greater than 0. If it were to let it be negative then it doesn't make sense in terms of a mean count.
- λ is a mean not a value that can be observed so it need not be an integer value.

Example

Maria is interested in the number of apples that will be sold at her fruit stand every Friday. She has seen that the mean number of apple sales is 19.3. What is the probability she will sell exactly 23 apples this Friday?

Here $x = 23$ and $\lambda = 19.3$:

$$
\begin{aligned}
f(23) = P(X = 23) &= \frac{\lambda^x e^{-\lambda}}{x!} \\
&= \frac{19.3^{23} e^{-19.3}}{23!} \\
&= \frac{3.69 \times 10^{29} 4.15 \times 10^{-9}}{2.58 \times 10^{22}} \\
&= 0.0594
\end{aligned}
$$

Notice how large some of the numbers can get when dealing with these calculations. Need to be careful when dealing with scientific notation.

Summary

The Poisson experiment and its associated random variable and probability distribution are a little more complex but extremely important.

- The sample space is quite large (countably infinite actually).
- The random variable is simply the number success across a specified time/space.
- The probability distribution can be used to find probabilities in this complex sample space.
- We will look at other properties of this type of experiment later.