Data Science 1
Probability
Independent Events

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Introduction

Recall the following:

- E is an event in a sample space $\mathcal{S}$.
- $P(\mathcal{S}) = 1$.
- $E \cup F$ is the union of events E and F.
- $E \cap F$ is the intersection of events E and F.
- Probability behaves like area.

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Two events E and F are said to be independent if both of these statements are true:

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We know $P(Score1) = P(Score2) = 0.43$ and $P(Score1 \cap Score2) = 0.172$.

$$\begin{aligned} P(Score1|Score2) &= \frac{P(Score1 \cap Score2)}{P(Score2)} \\ &= \frac{0.172}{0.43} \\ &= 0.4 \end{aligned}$$

Since $P(Score1|Score2) = 0.4 \neq 0.43 = P(Score1)$ the consecutive penalty shots are **not independent**.

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These rules are very helpful.

Summary

Independence is one of the most important ideas we have in probability and has huge impacts in statistical inference as well.

- Independence means $P(E|F) = P(E)$ and $P(F|E) = P(F)$.
- This gives the rule $P(E \cap F) = P(E)P(F)$.
- Many probability distributions depend on independence.
- Often independence must be **assumed**, which can be problematic.