



CPS Paper

Minimum information copula under fixed Kendall's rank correlation

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Submission ID: 737

Reference Number: 737

Presentation File

abstracts/ottawa-2023_dda16125a1b0f0819b3a2b3531b4a631.pdf

Brief Description

The minimum information copula (or the maximum entropy copula) is the most independent copula satisfying the given constraints.

For these constraints, first-order expectation constraints on moments, such as Spearman's rank correlation, are mostly considered.

On the other hand, such copulas under second-order constraints have not been studied well.

We present a variant of minimum information copula that has a constraint on a popular second-order constraint known as Kendall's rank correlation, instead of first-order constraints.

Due to this modification, the convexity of the problem becomes non-trivial and the form of density function of this variant is unknown.

We analyze its property via one of the widely known discrete approximation of copulas, called checkerboard copulas.

Checkerboard copulas can be considered identical to contingency tables.

First, we introduce a transfer operation of probability mass on checkerboard copulas, which is technically equivalent to considering non-orthogonal basis of the total space of checkerboard copulas.

Using this approach, we show several mathematical properties of the minimum information checkerboard copula under fixed Kendall's rank correlation.

Firstly, this copula is characterized by a certain amount, which we name as "extended log odds ratio".

It is also guaranteed that the density of this copula belongs to a function class known as "total positivity of order two (TP2)", one of the positive dependence properties that has been extensively studied for copulas.

Furthermore, geometric interpretations of this problem setting will be investigated.

Abstract

The minimum information copula (or the maximum entropy copula) is the most independent copula satisfying the given constraints. For these constraints, first-order expectation constraints on moments, such as Spearman's rank correlation, are mostly considered. On the other hand, such copulas under second-order constraints have not been studied well. We present a variant of minimum information copula that has a constraint on a popular second-order constraint known as Kendall's rank correlation, instead of first-order constraints. Due to this modification, the convexity of the problem becomes non-trivial and the form of density function of this variant is unknown. We analyze its property via one of the widely known discrete approximation of copulas, called checkerboard copulas. Checkerboard copulas can be considered identical to contingency tables. First, we introduce a transfer operation of probability mass on checkerboard copulas, which is technically equivalent to considering non-orthogonal basis of the total space of checkerboard copulas. Using this approach, we show several mathematical properties of the minimum information checkerboard copula under fixed Kendall's rank correlation. Firstly, this copula is characterized by a certain amount, which we name as "extended log odds ratio". It is also guaranteed that the density of this copula belongs to a function class known as "total positivity of order two (TP2)", one of the positive dependence properties that has been extensively studied for copulas. Furthermore, geometric interpretations of this problem setting will be investigated.