



IPS Paper

## Multilayer random dot product graphs: estimation and online change point detection

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### Brief Description

We begin by introducing the multilayer random dot product graph (MRDPG) model, which can be seen as an extension of the random dot product graph model to multilayer networks.

The MRDPG model is convenient for incorporating nodes' latent positions when understanding connectivity.

Modelling a multilayer network as an MRDPG, we further deploy a tensor-based method and demonstrate its superiority over the state-of-the-art methods.

Moving from a single MRDPG to a sequence of MRDPGs, we are concerned with an online change point detection problem in dynamic multilayer random dot product graph (D-MRDPG), especially with random latent positions.

At every time point, we observe a realisation from an  $L$ -layered MRDPG.

Across layers, we assume that common node sets and random latent positions are shared, but allow for different connectivity matrices.

Formulating the realisation as an adjacency tensor, the connectivity is characterised by an  $L$ -dimensional distribution.

Across time, we assume that the distribution sequence possesses an abrupt change.

Our ultimate goal is to detect this distributional change in an online fashion.

With control of false alarms, we aim to detect the change with minimal delay.

We devise a novel nonparametric change point detection algorithm, with a kernel estimator in its core, allowing for the case when the density does not exist.

An upper bound on the detection delay is derived, allowing for the node labels to vary across time and the model parameters, including the number of nodes, the dimension of latent position and the magnitude of the change, to vary as functions of the change point location.

Our theoretical findings are supported by extensive numerical experiments, with the code available online (<https://github.com/MountLee/MRDPG>).

### Abstract

We begin by introducing the multilayer random dot product graph (MRDPG) model, which can be seen as an extension of the random dot product graph model to multilayer networks. The MRDPG model is convenient for incorporating nodes' latent positions when understanding connectivity. Modelling a multilayer network as an MRDPG, we further deploy a tensor-based method and demonstrate its superiority over the state-of-the-art methods.

Moving from a single MRDPG to a sequence of MRDPGs, we are concerned with an online change point detection problem in dynamic multilayer random dot product graph (D-MRDPG), especially with random latent positions. At every time point, we observe a realisation from an  $L$ -layered MRDPG. Across layers, we assume that common node sets and random latent positions are shared, but allow for different connectivity matrices. Formulating the realisation as an adjacency tensor, the connectivity is characterised by an  $L$ -dimensional distribution. Across time, we assume that the distribution sequence possesses an abrupt change. Our ultimate goal is to detect this distributional change in an online fashion. With control of false alarms, we aim to detect the change with minimal delay. We devise a novel nonparametric change point detection algorithm, with a kernel estimator in its core, allowing for the case when the density does not exist. An upper bound on the detection delay is derived, allowing for the node labels to vary across time and the model parameters, including the number of nodes, the dimension of latent position and the magnitude of the change, to vary as functions of the change point location. Our theoretical findings are supported by extensive numerical experiments, with the code available online (<https://github.com/MountLee/MRDPG>).