Urging a Paradigm Change:
Why and How to Train Introductory Statistics Students in Bayesian Thinking?

Dalene Stangl

Life is a sequence of decisions and actions based upon data-driven beliefs. If so, why do we not teach students to navigate this decision making sequence in a coherent way, by setting up structures to seamlessly move from belief and values elicitation to data generation, analysis, and action? Instead we have chosen to narrowly focus on statistical inference in a hypothesis testing framework, we silo our impact into a narrow scope. Thinking as Bayesians lets us widen our scope to much broader decision making. Unless we adapt to this wider scope of decision making, our field may find itself extinct.

This article begins discussion on why and how Bayesian foundations and methods should be taught to students in primary and secondary schools as well as university undergraduates in all disciplines. I'll start with some early work on the subject and provide some personal views on where we should be going. I'll end with some suggestions for how the statistics education community can be more active and effective in changing the attitudes that have been to date resisted.

What is Education?

David Hansen – Professor and Director of the Philosophy and Education Program at Teachers College, Columbia University. He has a wonderful speech titled "Is Education Possible Today?" The entire talk discusses why the answer to that question is not obvious and explains why the question is more important than any answer or response. For transformative education, it is the necessity of the question and the importance of keeping the question open. One cannot rush thinking, instead we must follow the course why the answers are not "the answer." Education is about "keeping questions open." In this paper I propose that statistics education (the first course) while being transitioned away from a "math-heavy" subject to a "computer science-heavy" subject, should also be transitioning to a "philosophy-heavy" subject. A subject of how is it via quantitative information and thinking, that we can come to know what we know and make decisions based on it?

Hansen describes that in the world, millions of us, serious-minded educators, spend trillions of hours teaching every year with various levels of success in "transformational" experiences of our students. What keeps us from succeeding? Hansen argues that national systems, conditions of political & economic & moral will to realize such experiences do not exist – imperatives and reward structures of a globalizing economic order have reduced education into a courtesy call that amounts to shaping people to fit that economic system. Lifelong learning is morphed into credentialing. Life-long-learning is nothing more than people internalizing this shaping function, so one is left continually adapting to the system rather than questioning it. Isn't this exactly what has happened to statistics education and our persistent focus on testing and p-values?

Given these structures, why and how do we continue to show up? How does one traverse the path between head in the sand and cynic? Hansen argues that by being realistic, and honorable, and pragmatic. By authentically reaching out for education. Stay with the question! Apprehend and feel the question as fully as possible. Take the philosophical approach – what is knowledge vs belief ?

Hansen's answer to how do we stay within the question "Is Education Possible Today?"-- stay as near as possible to education, but strip away much of what we consider education. If strip what's left?

- It is listening and contemplative silence
- Resist answers that come even from our own priors

- Doesn't imply absence of action
    - Special form of actions – bring to the table priors and utility functions of others
    - Teacher goal: disagreement is natural – tensions are OK
    - Living awareness of asking a question may cause suffering
        - Anguish is felt when one discovers their personal views alienates from one's community
        - Anguish when sense of open questioning conflicts with what others in the system will support
    - True education is provocative: It leads to pain, doubt, confusion
    - True education resist the YES/NO to the "sometimes", it is an invitation to philosophy – an invitation to open ended questions, it resists premature, impatient resolution to a question

System Pressures

- Questions – In stats education – biggest challenge is "other disciplines view us as a set of right/wrong steps, and algorithms followed to a right/wrong answer rather than a way of disciplined thinking, challenging, telling a story, and peruasive argument
- Standardized testing – Is it education?
- Pressure for efficiency
- Pressure for measurable outcomes? We measure what is easy to measure – vicious cycle of teaching what's easy to measure

Young people are clamoring that they want to be or create disruptors. A disruptor is an innovation that creates a new market and value network and eventually disrupts an existing market and value network, displacing established market leading firms, products and alliances.

Ellevate – 1000s of women were asked what industry most needs disruption? #1 answer was Education 25% (Health Care 22%, Finance 13%, Environment 10%, All others <10%)

How We Currently Teach

At the graduate and advanced undergraduate level the last thirty years has experienced dramatic change in the teaching of statistics. At these education levels statistical computation and Bayesian thinking is now embedded in the curriculum. Unfortunately this is not yet the case in primary and secondary education or in undergraduate education of non-statistics majors. However, now even in traditional statistics education circles, the ground is rumbling. In a 2015 paper, George Cobb urged us to flatten our prerequisites and teach through research. He argues that the reliance on technical mathematics led us to think we could only teach Bayesian thinking to students who could do multidimensional integrals for the denominators of Bayes' theorem. Well that's not true of everyone. A handful of Bayesians thinkers have been arguing for nearly twenty-five years that this is not the case and that the lack of effort on our part to reach these groups is a mistake.

Twenty years ago Don Berry and Jim Albert argued that it was important to teach Bayesian methods to undergraduates who were not majoring in statistics. They argued against those who proposed that Bayesian methods could not and should not be taught at this level. (Albert, 1997; Berry, 1997; Moore, 1997). Arguments against teaching Bayesian methods to these groups included notions that students could not understand conditional probability, that there was a lack of consensus among Bayesians on how to tackle standard problems, that Bayesian methods were rarely used, and that the inclusion of Bayesian ideas would impede trends toward use of real data. While progress over the last twenty years further counters these arguments, these views continue to be held by almost all in the statistics education community.

Bayesian thinking, i.e., to learn to collect information and merge that information with our values to make coherent decisions, should indeed be the ultimate goal of educating the masses. While the importance of statistical understanding in primary and secondary education has continued to grow (~200,000 students took the advance placement (AP) statistics exam in 2016), there has been no incorporation of Bayesian thinking into the AP curriculum. Inclusion of Bayesian thought in undergraduate courses for nonmajors is also nearly absent. Opposition to teaching Bayesian statistics to these groups remains strong, and ISBA and SBSS have been rather absent in this fight although several of its members have been pushing for years.

The statistics education community, and its primary ring-leaders are not Bayesian. The International Conference on Teaching Statistics (ICOTS) and US Conference on Teaching Statistics (USCOTS) usually has but a smattering of sessions on teaching Bayesian statistics. It's barely on their radar. Jim Albert, Bill Bolstad, Peter Sedlmeir, I, and a few others have continued to push the teaching of Bayesian methods at this level through participation at USCOTS and ICOTS, but this is not an easily moved contingent. These conferences are a pretty lonely place for the few Bayesians that attend.

George Cobb's 2015 paper on rethinking the undergraduate curriculum describes "*the tension between the ad hoc pragmatism of context-based courses and the aesthetic unity of mathematically-based courses.*" He also discusses the ever increasing gap between our half-century-old curriculum and our contemporary statistical practice that increasingly relies on Bayesian as opposed to frequentist methods. He laments how our dependence on mathematics has made our education inaccessible to many and discusses how computing can help open up new possibilities. He discusses how the computer, rather than logic, has persuaded our profession to embrace Bayes, and within his educational imperatives includes the teaching of all probability as conditional probability and teaching more complicated probabilities via approximation and simulation. He argues that these two changes remove the major impediments to teaching Bayesian inference at the elementary level.

It is not just statistics education conferences where Bayesians and Bayesian ideas are absent. The AP statistics exam is completely frequentist. Popular textbooks, if lucky, include Bayes' theorem, but rarely include other Bayesian foundations or inference topics. There were no Bayesians on the American Statistical Associations committee to revamp the GAISE guidelines. The statistics education community's current "big thing" is randomization and permutation tests for teaching statistics. While this may help students better understand the concept of a sampling distribution, it still puts undo emphasis on hypothesis testing and is unhelpful at making a p-value a meaningful measure of evidence. It does nothing to train students to think in a more decision-theoretic way.

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Perhaps a more justified argument against teaching Bayesian ideas at this level would be that there is still few teaching resources for this audience. There is Berry's elementary statistics book, *Statistics: A Bayesian Perspective* (1996), that is well suited for audiences without calculus backgrounds. His book relies only on high school algebra and remains my go-to reference for any non-statistician who asks me for a first book. While recommending the book, it still needs two things, a facelift and a follow-on book that would take the reader into the analysis of data, and this is where it gets tricky. We have Tony O'hagens's First Bayes and Jim Albert's Minitab macros and javascript software. We have Kruske's Bayes with R, JAGs and STAN. But what about the students in high schools and community colleges who are still stuck with hand calculators? What about the students at universities with access only to SPSS, JMP, SAS, STATA and other software?

There is also the question of what would we teach in courses for majors other than statistics. Albert (2001) describes a "Statistics for Poets" class. This class focuses on the distinction between statistics and parameters, the inherent variability in data, that sample data provide an incomplete description of the population, the dependence of statistical procedures on the underlying assumptions of the model, the distinction between inference procedures including estimation, testing, prediction, and decision making, and the interpretation of statistical "confidence." He argues that the primary advantage of teaching from a Bayesian viewpoint is that Bayes' thinking is more intuitive than the frequentist viewpoint and better reflects the commonsense thinking about uncertainty that students have before taking a statistics class. Albert and Rossman (2001) have published a collection of activities that assist in teaching statistics from a Bayesian perspective. Like the Berry textbook, it covers estimating proportions and means using discrete and continuous models. Web-based javascript software to illustrate probability concepts and perform Bayesian calculations is also available.

Examples

Discrete world – Stangl examples – binomial data, discrete prior
Conjugate world – Stangl examples – beta-binomial, normal-normal
Simulations – Cobb example
MCMC – Witmer example

What can we do?
To break into statistics education at the primary and secondary levels, we must crack the intro college course for nonmajors. This course drives the AP curriculum, which in turn drives the primary and secondary curricula. Here are some first steps.

1. Create an ISBA and SBSS Teaching Section aimed at education of students in primary and secondary school and undergraduates majoring in disciplines other than statistics.
2. Create a teaching resources repository that houses materials that teachers can freely and openly access.
3. Create a MOOC that models an undergraduate statistics course for non-majors. (Mine Cetinkaya Rundel, David Banks, Merlise Clyde and Colin Rundel from Duke, Herbie Lee from UC Santa Cruz run such courses on Coursera, but both courses are listed as "intermediate".)
4. Work on a facelift of the Berry textbook and write a textbook that takes students a step further.
5. Create a subgroup of the Teaching Section that petitions textbook companies and the College Board to include Bayesian thinking.

Teaching Resources Repository

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