AN ALTERNATIVE ERROR INNOVATION UNDER ASYMMETRIC GARCH MODELS WITH APPLICATION

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ABSTRACT

This paper attempts to develop two asymmetric GARCH innovations. An empirical analysis of the mean return and conditional variance of Nigeria Stock Exchange (NSE) index is performed using various error innovations in GARCH models. The parameter estimations are carried out on the assumption of normality and non- normality of GARCH innovations. Our result shows that GARCH(1,1) and APARCH(1,1) models with anomalous densities improves overall estimation for measuring conditional variance. Log-likelihood and Model Selection Criteria is used to determine the robust model. The prediction performance of these conditional changing variance models is compared using Root Mean Square Error(RMSE) and Mean Absolute Percentage Error (MAPE). Generalized Length Biased Scaled-t Innovation using APARCH(1,1) model is the most robust for forecasting Nigeria Stock Exchange Index.

Keywords: NSE index, Generalized Length Biased Scaled -t distribution, Generalized Beta Skewed -t distribution , GARCH

1.0 INTRODUCTION

GARCH models have been developed to account for empirical regularities in financial data. Many financial time series have a number of characteristics in common; asset prices are generally non stationary while returns are usually stationary. Some financial time series are fractionally integrated. Return series usually show no or little autocorrelation. Serial independence between the squared values of the series is often rejected pointing towards the existence of non-linear relationships between subsequent observations. Volatility of the return series appears to be clustered. Normality has to be rejected in favor of some thick-tailed distribution. Some series exhibit so-called leverage effect that is changes in stock prices tend to be negatively correlated with changes in volatility. A firm with debt and equity outstanding typically becomes more highly leveraged when the value of the firm falls. This raises equity returns volatility if returns are constant. Black(1976), however, argued that the response of stock volatility to the direction of returns is too large to be explained by leverage alone.

In his seminal paper, Engle (1982) proposed to model time-varying conditional variance with Autoregressive Conditional Heteroskedasticity (ARCH) processes using lagged disturbances; Empirical evidence based on his work showed that a high ARCH order is needed to capture the dynamic behaviour of conditional variance. The Generalized ARCH (GARCH) model of Bollerslev (1986) fulfils this requirement as it is based on an infinite ARCH specification which reduces the number of estimated parameters from infinity to two.

Another problem encountered when using GARCH models is that they do not always fully embrace the thick tails property of high frequency financial times series. To overcome this drawback

Bollerslev (1987), Baille and Bollerslev (1987) and Beine et al (2002) have used the Student's t-distribution. Similarly to capture skewness Liu and Brorsen (1995) used an asymmetric stable density.

The disadvantage of the normal GARCH (1,1) model is that the conditional excess kurtosis is zero, both unconditional and conditional skewness are zero, thus, volatility clustering , leverage effect and leptokurtosis cannot be capture adequately. This work intends to re-modify error distributions of GARCH (p, q) model inference under violation of normality in favour of some non-normal distributions.

2.0 METHODOLOGY

The data consist of 1974 observations of the NSE Stock Index from period January 2000 to December 2015 which was obtained from statistical Central Bank of Nigeria Bulletins 2016. To estimate and forecast this index, we use GARCHFIT in R package. Initially the assets prices are transformed into log return series in equation 1 and R_t in equation 2 is the Autoregressive model is estimated for the return series. Where Y_t is All Share Index (ASI) for day t.

$$R_t = \log Y_t - \log Y_{t-1} = \log\left(\frac{Y_t}{Y_{t-1}}\right) = \log\left(1 + \frac{Y_t - Y_{t-1}}{Y_{t-1}}\right) \tag{1}$$

$$R_t = a_0 + \sum_{i-1}^{s} a_i R_{t-i} + \varepsilon_t \tag{2}$$

The error terms follows Normal , Student-t , GED, new proposed Generalized length biased scale- t and Generalized Beta Skew –t Distributions. We also consider GARCH (1,1) and APARCH (1,1) model as the variant models stated in equation 3 and 4 respectively

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3}$$

Where $\alpha_0 > 0$, $\alpha_i \geq 0$ (for i=1, ---, q) , $\beta_j \geq 0$ (for j=1, …, p) is sufficient for conditional variance to be positive.

$$\sigma_t^2 = \alpha_0 + \alpha_i \left(\left|\varepsilon_{t-1}\right| - \gamma \varepsilon_{t-1}\right)^2 + \beta_j \sigma_{t-1}^2 \tag{4}$$

Where $\alpha_0 > 0$, $\alpha_i \geq 0$ (for i=1, ---, q) , $\beta_j \geq 0$ (for j=1, …, p) and $-1 < \gamma_i < 1$ i= 1, . . ., p allows for asymmetric parameter.

The generalized length biased distribution is derived when the weighted function depend on the length of units of interest (i.e. w(y) = y) , then the length biased Scaled- t distribution is given as :

$$g(y) = \frac{yf(y)}{\mu} \tag{5}$$

Where the probability function of student –t distribution is

$$f(y) = \frac{\sqrt{\frac{v+1}{2}}}{\sqrt{\frac{v}{2}}\sqrt{\pi(v-2)\sigma^2}}\left[1 + \frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)} \tag{6}$$

Substitute equation (6) into (5), that gives

$$g(y) = \frac{\sqrt{\frac{v+1}{2}}}{\mu\sqrt{\frac{v}{2}}\sqrt{\pi(v-2)\sigma^2}}\, y\left[1 + \frac{(y-\mu)^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)} \tag{7}$$

If we consider the variance equation to be GARCH (1,1) model, mean equation to be AR(1) and the error term to be generalized length biased scaled-t distributed, then the log likelihood is

$$L(\theta)=n\log\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\,\Gamma\left(\frac{v}{2}\right)}-\frac{1}{2}\sum_{t=1}^{n}\log\sigma_t^2-\left(\frac{v+1}{2}\right)\sum_{t=1}^{n}\left[\log\left(1+\frac{\varepsilon_t^2}{\sigma_t^2(v-2)}\right)\right]+\sum\log\left[\alpha_0+\alpha_1 y_{t-1}+\varepsilon_t\right]-n\log\mu \tag{8}$$

The generalized beta distribution of the first kind was introduced by McDonald (1984), with link function

$$g(y)=\frac{c}{\beta(a,b)}\left[F(y)\right]^{ac-1}\left[1-F(y)^c\right]^{b-1}f(y) \tag{9}$$

Where $F(x)=I_{x(abc)}$ is an incomplete beta function and f(y) is the probability function of student-t.
A random variable y is said to be a Generalized Beta Skewed –t distribution if

$$f(y:a,b,c)=\frac{c}{\beta(a,b)}I^{ac-1}\left[1-I^c\right]^{b-1}\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sigma\,\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)}}\left[1+\left(\frac{y-\mu}{\sigma}\right)^2\frac{1}{v-2}\right]^{-\frac{v+1}{2}} \tag{10}$$

If we assume that $\varepsilon_t\sim GBt(v,\mu,\sigma,a,b,c)$, we have

$$f(y_t:a,b,c)=\frac{c}{\beta(a,b)}I^{ac-1}\left[1-I^c\right]^{b-1}\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma_t^2}}\left[1+\frac{\varepsilon_t^2}{\sigma_t^2}\frac{1}{v-2}\right]^{-\frac{v+1}{2}} \tag{11}$$

Considering mean equation as AR (1) model and variance equation as APARCH (1,1) model then the log likelihood when the error term follows generalized beta Skew t distribution;

$$l=n\log c-n\left[\log\Gamma a+\log\Gamma b-\log\Gamma(a+b)\right]+n\log\Gamma\left(\frac{v+1}{2}\right)-n\log\Gamma\left(\frac{v}{2}\right)-\frac{n}{2}\left[\begin{array}{l}\log\pi+\log(v-2)+\\ \sum_{t=2}^{n}\log\left[\alpha_0+\alpha_1\left(\left|\varepsilon_{t-1}\right|-\gamma_1\varepsilon_{t-1}\right)^2\right]\end{array}\right]+$$

$$(ac-1)\sum_{t=2}^{n}LogI+(b-1)\sum_{t=2}^{n}\log(1-I^c)-\left(\frac{v+1}{2}\right)\sum_{t=2}^{n}\log\left[1+\frac{\left(Y_t-\phi_1 Y_{t-1}\right)^2}{\alpha_0+\alpha_1\left(\left|\varepsilon_{t-1}\right|-\gamma_1\varepsilon_{t-1}\right)^2(v-2)}\right] \tag{12}$$

3.0 RESULTS AND DISCUSSION

To obtain a stationary series, we use the returns $R_t=100\left(\log\left(Y_t\right)-\log\left(Y_{t-1}\right)\right)$ where Y_t is the closing value of index at month t. The sample statistics for the returns R_t are exhibited in table 1 for Nigeria Stock Exchange (NSE) index (sample January 2000 to December 2015), the skewness is negatively skewed and also exist fourth moment showing negative kurtosis which indicate non-noraml distribution. Shapiro-Wilk test indicate non normality. While time plot of series is stationary series after transformation, Autocorrelation function (ACF) and Partial Autocorrelation Function (PACF) point towards AR(1) model.

Table I: **Descriptive statistics for Returns**

Index	Min	Median	Mean	Max	Skewness	Kurtosis	Shapiro –Wilk test
NSE	1.00	96.50	96.40	191.00	-0.00763	-1.2260	0.9538

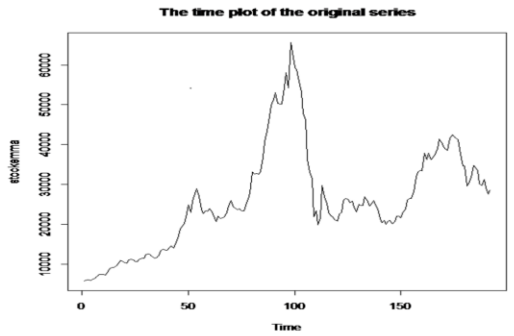

Figure 1: The time plot of the original series

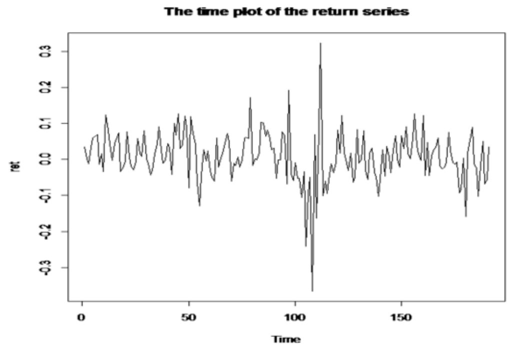

Figure 2: The time plot of the return series

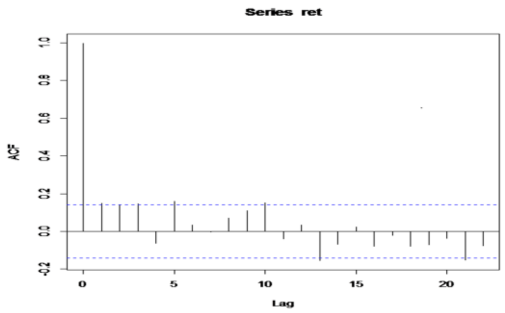

Figure 3: Autocorrelation Function plot

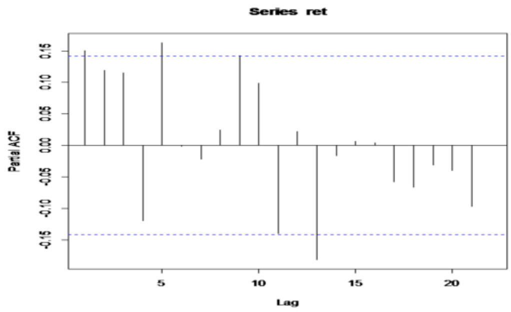

Figure 4: Partial Autocorrelation Function plot

The basic estimation model consist of two equations, one for the mean which is a simple autoregressive AR(1) model and another for the variance which is identified by a particular ARCH specification i.e. GARCH (1,1). For NSE index the models are estimated using R code by the approximate quasi- maximum likelihood estimator assuming normal, , student t, GED, Generalized Length Biased Scale-t and Generalized Beta Skewed-t as innovations. The Alkaike Information Criterion and Log-likelihood values from table II reveal that APARCH model better estimate the series than the traditional GARCH, to compare the different densities with model we apply the Akaike Information Criterion (AIC) and the log likelihood values. When we analyze the densities we find that Generalized length biased scaled-t and beta skewed-t distribution clearly out performed the normal distribution. Indeed the log likelihood function increases when using that length biased scaled-t distribution, leading to AIC criteria of 7.855 and 10.413 for normal density .

Table II: Model Selection Criteria Table For Existing And New Proposed Innovations

GARCH TYPE	LOGLIKE LIHOOD	AIC
GARCH-NORM	-995.69	10.4134
GARCH-STD	-1002.17	10.4913
GARCH-GED	-953.46	9.98394
GARCH-BST	-1040.15	**10.00341**
GRACH-LGST	-749.15	**7.9878**
APARCH-NORM	-995.48	10.43205
APARCH-STD	-1002.01	10.51055
APARCH-GED	-955.10	10.02183
APARCH-BST	-1040.15	**10.0002**
APARCH-LGBST	-749.15	**7.8557**

Table III: Forecasting Analysis for the NSE Index: Comparing between densities

MODEL	RMSE	AMAPE
GARCH-NORM	0.7565	0.7565
GARCH- STD	0.3899	0.3421
GARCH- GED	0.6261	0.6840
GARCH-BST	**0.3094*****	**0.3005*****
GARCH-LBST	**0.2912*****	**0.2900*****
APARCH-NORM	0.5379	0.8936
APARCH- STD	0.3499	0.3421
APARCH- GED	0.4567	0.4790
APARCH-BST	0.3168	0.3297
APARCH-LBST	**0.2809***	**0.2800***

The forecasting we obtain are evaluated using Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) predicting 24 steps ahead. The forecasting is reported by ranking the different models with respect to RMSE and MAPE for NSE index, the result support the asymmetric

APARCH than the GARCH model. The Generalized Length biased Scaled-t- APARCH(1,1) is the most successfully in forecasting NSE conditional variance.

4.0 CONCLUSION

The forecasting performance of GARCH (1,1) model was compared using different distributions for Nigeria Stock Index returns. We found that the new proposed Generalised Length biased APARCH(1,1) model is the most promising for characterizing the dynamic behaviour of these returns . For further studies, the newly proposed innovations can be investigated in other asymmetric models like IGARCH, FIGARCH, EGARCH ,regime switch GAS etc

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