Statistical Inference on the Two- and Three-State Availability of Repairable Units with the Sahinoglu-Libby Model

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ABSTRACT

With the advances in pervasive computing and wireless networks, quantitative risk measurement of component (unit) and network availability has become a challenging task. It is widely recognized that the forced outage ratio (FOR) of an imbedded hardware component is defined as the failure rate divided by the sum of the failure and repair rates; or FOR is the non-operating time divided by the total exposure time. However, it is also well documented that FOR is not a constant a random variable. The probability density function (p.d.f.) of the FOR is the Sahinoglu-Libby (SL) probability model, used if certain underlying assumptions hold. The failure and repair rates are taken to be the generalized gamma variables where the corresponding shape and scale parameters respectively are not equal. The SL model is shown to default to that of a standard two-parameter beta p.d.f. when the shape parameters are identical. Decision theoretic solutions are sought to compute small-sample Bayesian estimators by using informative and non-informative priors for the failure and repair rates with respect to three definitions of loss functions. These estimators for component availability are then propagated to calculate the network expected source-target availability for simple complex networks. On the other hand, an often overlooked fact is that many real-life grid units from routers or servers in cybersystems to electric-power generating plants, and water-supply networks or dams do not operate in a dichotomously full or empty capacity. Due to lack of a closed-form solution of the DFOR in the three-state model as opposed to closed-form of the two-state model, the analysis can be conducted by Monte Carlo simulations using the empirical Bayesian principles to estimate the full and derated availability of a repairable hardware unit. Industrial applications for units will be numerically illustrated. For the three-state model following the Monte Carlo simulations, authors will show how to estimate the resultant p.d.f.s obtained from numerical analyses regarding single units.

Keywords: Two-State, Three-State, FOR, DFOR, Sahinoglu-Libby p.d.f., Bayesian Estimation, Monte Carlo Simulation, Gamma, Beta

1. INTRODUCTION

The probability density function (p.d.f.) of the FOR was earlier studied in a textbook by the primary author, designated the Sahinoglu-Libby (SL) probability model used if certain underlying assumptions hold (Sahinoglu, 2007 and Sahinoglu *et al*., 2005). The failure and repair rates were taken to be the generalized gamma variables where the corresponding shape and scale parameters respectively were not equal. The SL model was shown to be default to that of a standard two-parameter beta p.d.f. when the shape parameters are identical. The method proposed was superior to estimating availability by dividing total uptime by exposure time. Examples had shown the validity of this method to avoid over- or underestimation of availability when only small samples or insufficient data exist for the historical life cycles of units. In this paper, however, additionally we shall also study a three-state SL similar to the two-state SL, which will be called three-state Sahinoglu-Libby model (Sahinoglu, 2007 and Sahinoglu *et al*., 2005). Due to infeasibility of closed-form solutions, the analysis will be carried out using Monte Carlo simulations, obeying the principles of Bayesian principles similar to Chapter 5 of the author‟s textbook (Sahinoglu, 2007). In studying large capacity production units, it is necessary to consider the probabilities associated with one or more forced derated states rather than accepting the unit being either available or unavailable (Billinton, 1970). This real-life situation has been also studied in a publication (Sahinoglu *et al*., 1983) where a system unit was assumed to exist in one of the three states; that is, UP, DERATED and DOWN. Following the Monte Carlo simulations, analytical p.d.f.s will be approximated.

2. METHODS

2.1 Two-State Sahinoglu-Libby Probability Model

In using the distribution function technique, the p.d.f. of FOR = $q = \lambda/(\lambda+\mu)$ is obtained first by deriving its c.d.f. $G_Q(q) = P(Q \leq q) = P(\lambda/(\lambda+\mu) \leq q)$ and then taking its derivative to obtain $g_Q(q)$ as in equations 5A.1 to 5A.18 in Appendix 5A (Sahinoglu, 2007, pp. 26-32) and in reference (Sahinoglu *et al*., 2005, p. 1487):

$$g_Q(q) = \frac{\Gamma(a+b+c+d)}{\Gamma(a+c)\Gamma(b+d)}\ \frac{(\xi+x_T)^{a+c}(\eta+y_T)^{b+d}(1-q)^{b+d-1}q^{a+c-1}}{[\eta+y_T+q(\xi+x_T-\eta-y_T)]^{a+b+c+d}}$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-q)^{\beta-1}q^{\alpha-1}\ \left\{\ \beta_1{}^{\alpha}\ \beta_2{}^{\beta}\ \left[\ \frac{1}{\beta_2+q(\beta_1-\beta_2)}\ \right]^{\alpha+\beta}\ \right\}$$

$$= B(\alpha,\beta)(1-q)^{\beta-1}q^{\alpha-1}\left[\ \frac{\beta_2}{\beta_2+q\beta_2(L-1)}\ \right]^{\alpha+\beta}L$$

$$=B(\alpha,\beta)(1-q)^{\beta-1}q^{\alpha-1}\left[\ \frac{1}{1+q(L-1)}\ \right]^{\alpha+\beta}L \tag{1}$$

where,

$$B(\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \tag{2}$$

Note that $g_Q(q)$ is the p.d.f. of the random variable Q = FOR, where α = a + c, $\beta = b + d$, $\beta_1 = \xi + x_T$, and $\beta_2 = \eta + y_T$; and $0 \le q \le 1$. If $L = \beta_1 / \beta_2 = 1$ or $\beta_1 = \beta_2$, the usual two-parameter beta p.d.f. is obtained. An alternative original derivation of the same p.d.f. termed under generalized multivariate beta distribution is given by Libby in the same year 1981 (Sahinoglu *et al*., 2005). The expression in Equation (1) can also be reformulated in terms of SL($\alpha = a + c$, $\beta = b + d$, L = β_1 / β_2), as follows:

$$g_Q(q) = \frac{L^{\alpha+c} q^{\alpha+c-1} (1-q)^{b+d-1}}{B(b+d, a+c)[1-(1-L)q]^{a+b+c+d}} \tag{3}$$

where

$$B(b + d, \mathrm{a} + c) = \frac{\Gamma(a+c)\Gamma(b+d)}{\Gamma(a+b+c+d)}, \text{ and } L = \frac{\xi + x_T}{\eta + y_T} \tag{4}$$

Note, if $L = 1$, the Sahinoglu-Libby p.d.f. reduces to a Beta(α , β) p.d.f.

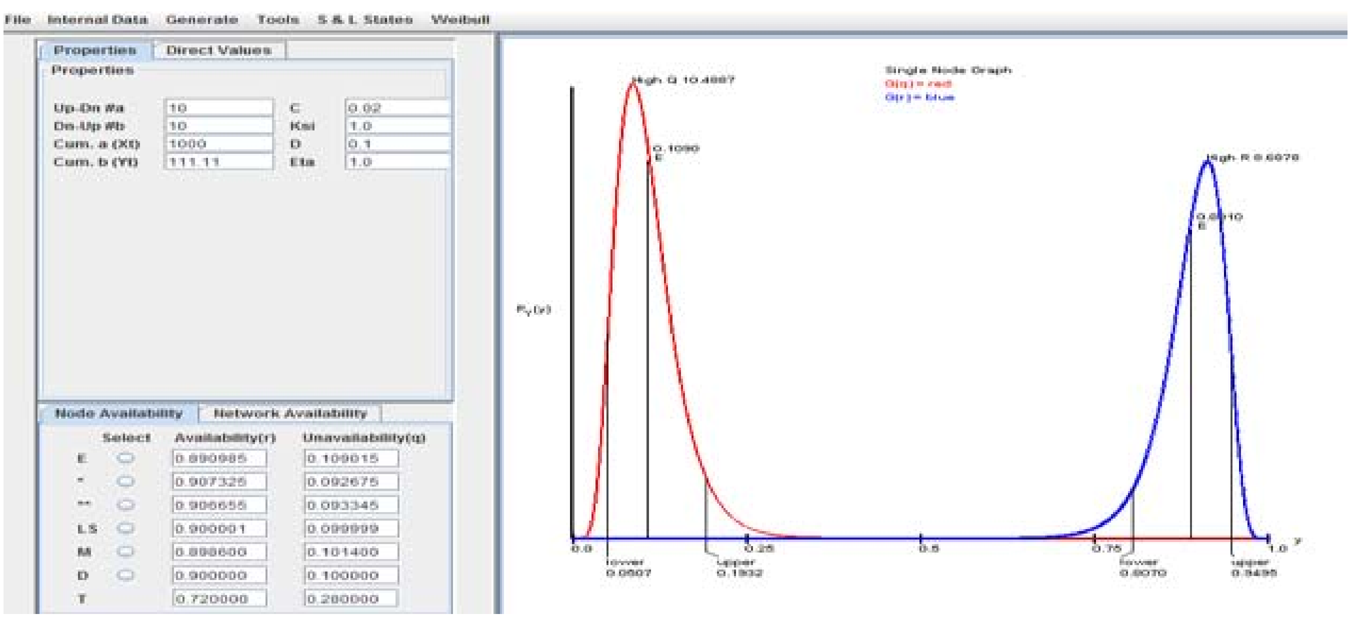

Fig. 2.1.1: Given the input table, the p.d.f. of the two-state SL is plotted for up (r) and down(q) for a 90% confidence analytically showing mode (m), mean (E) with upper & lower confidence bounds.

2.2 Three-State Sahinoglu-Libby Probability Model

In studying large capacity generation (power) or production (cyber) units, it may be necessary to consider the probabilities associated with one or more forced derated-outage states rather than considering the unit as being either available or unavailable (Sahinoglu *et al*., 2005). In summary, there are gray areas or in-between capacities which are called derated states. However in this research paper, we will only consider a single derated state rather than multiple ones, which may well exist in practice such as in 50%, 60% or 75% derated capacity. But now, we have not only full-FOR but also derated-FOR or DFOR, that will be equal to derated operating time over the total exposure time. DFOR = Derated time/(Up time + Derated time + Down time). However, it is also well documented that any FOR or DFOR is not only a constant but a random variable (Sahinoglu *et al*., 2005). The probability density function (p.d.f) of the FOR was earlier studied to be a (Bayesian) Sahinoglu-Libby (SL) probability model, where certain underlying assumptions hold. However, we shall study above and beyond a traditional two-state SL, namely the three-state probability model where the transition rates are Gamma distributed (See Section 2.2.1 to 2.2.6). Let us examine the related state space diagram (Billinton, 1970):

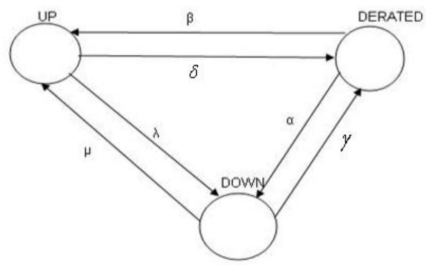

Figure 2.2.1: General Three-State Diagram of a repairable hardware unit with Up, Down and Derated.

Let: λ =transition rate from up to down (forced out) state; μ = transition rate from down to up state; δ = transition rate from up to derated (partially forced out) state; β = transition rate from derated (partially forced out) to up state; α = transition rate from derated (partially forced out) to down (forced out) state; γ = transition rate from down (forced out) to derated (partially forced out) state. Utilizing an important reference (Sahinoglu *et al*., 2005, p.156-7) and adjusting it to our design by converting to the Greek letters from Latin, the time dependent (assuming negative exponential densities for each state‟s sojourn time go to a steady state) probabilities of occupying one of the three states are given in skipping the intermediate steps of derivation where DENOM= $\mu\beta + \mu\gamma + \alpha\beta + \lambda\beta + \lambda\gamma + \delta\mu + \delta\alpha + \lambda\mu + \delta\gamma$

$$P(UP) = FOR= \frac{\mu\beta + \mu\gamma + \alpha\beta}{DENOM} \tag{5}$$

$$P(DERATED) = DFOR= \frac{\delta\mu + \delta\alpha + \lambda\mu}{DENOM} \tag{6}$$

$$P(DOWN) = 1- P(UP) – P(DERATED) = \frac{\lambda\beta + \lambda\gamma + \delta\gamma}{DENOM} \tag{7}$$

A closed form solution of the three-state SL is out of question, and infeasible to obtain with too many variables. We will therefore have to simulate the above formulated P(UP), P(DER) and P(DOWN) by generating Monte Carlo simulated deviates of the state transition rates. This will be done through deriving first the conditional posterior densities of the six transition rates from sections 2.2.1. to 2.2.6, and using random uniforms for generating the transitions that constitute the probabilities.

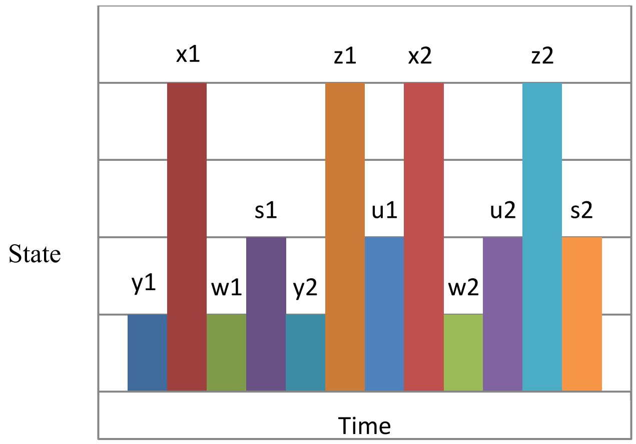

Figure 2.2.2: An Illustration of Feasible Transitions from Figure 1. 1 applied to Sections 2.2.1. to 2.2.6

2.2.1 Up to Down Failure Transition Rate (λ), e.g. from x1 to w1, or x2 to w2 in Fig. 2.2.2.
Let, *a* = number of occurrences of up (operating) times before going down (debugging)

$X_i = \sim \lambda e{-}\lambda X$

$x_T = \sum_{1}^{a} X_i$ total up (operating) times before going down (recovery) for *a* # such occurrences

λ = full {up to down} failure rate
c= shape parameter of gamma prior for full failure rate λ
ξ = inverse scale parameter of gamma prior for full failure rate λ

Now let the failure rate, λ have a gamma prior distribution:

$$\theta_1(\lambda) = \frac{\xi^c}{\Gamma(c)} \lambda^{c-1} \exp(-\lambda\xi),\ \lambda > 0 \tag{8}$$

The joint likelihood of the up-time random variables is

$$f(x_1,x_2,\ldots,x_a|\lambda)=\lambda a\exp(-xT\lambda), \tag{9}$$

The joint distribution of data and prior becomes:

$$k(\underline{x},\lambda) = f(x_1,x_2,\ldots,x_a,\lambda) = \frac{\xi^c}{\Gamma(c)} \lambda^{a+c-1} \exp[-\lambda(x_T+\xi)] \tag{10}$$

Thus, the posterior distribution for λ is

$$h_1(\lambda \mid \tilde{x} = \frac{k(\tilde{x},\lambda)}{\int_\lambda f(\tilde{x},\lambda)d\lambda} = \frac{\xi^c}{\Gamma(c)} \lambda^{a+c-1} \exp[-\lambda(x_T+\xi)] \div \frac{\xi^c}{\Gamma(c)}(x_T+\xi)^{-1}\Gamma(a+c) = \frac{1}{\Gamma(a+c)}$$
$$(x_T+\xi)\lambda^{a+c-1}\exp[-\lambda(x_T+\xi)], \tag{11}$$

which is also a Gamma [a+c, $(x_T+\xi)^{-1}$].

2.2.2 Down to Up Recovery Transition Rate (μ), e.g. from y1 to x1, or y2 to z1 in Fig. 2.2.2.
Let, *b*= number of occurrences of down (debugging) times before going up (operating)

$Y_i \sim \mu e{-}\mu^Y$

$y_T = \sum_{1}^{b} Y_i$ total down (recovery) times before going up for *b* # such occurrences

μ = full recovery (down to up) rate
d = shape parameter of Gamma prior for full recovery rate μ
η = inverse scale parameter of gamma prior for full recovery rate μ

Now let the full recovery rate, μ have a gamma prior distribution:

$$\theta_2(\mu) = \frac{\eta^d}{\Gamma(d)} \mu^{d-1} \exp(-\mu\eta), \ \mu > 0 \tag{12}$$

The joint likelihood of the down-time random variables is

$$f(y_1, y_{2,\ldots,} y_a | \mu) = \mu^b \exp(-y_T \mu) \tag{13}$$

The joint distribution of data and prior becomes:

$$k(\underline{y}, \mu) = f(y_1, y_{2,\ldots,} y_b, \mu) = \frac{\eta^d}{\Gamma(d)} \mu^{b+d-1} \exp[-\mu(y_T + \eta)] \tag{14}$$

Thus, similarly the posterior distribution for μ is

$$h_2(\mu | \tilde{y}) = \frac{1}{\Gamma(b+d)} (y_T + \eta)\mu^{b+d-1} \exp[-\mu(y_T + \eta)], \tag{15}$$

which is also a Gamma [b+d, $(y_T + \eta)^{-1}$].

2.2.3 Up to Derated Failure Transition Rate (δ), e.g. from z1 to u1, or z2 to s2 in Fig. 2.2.2.

Let, *o* = number of occurrences of up times before going derated

$z_T = \sum_{1}^{o} Z_i$ total up times before going derated for *o* # of such occurrences

$Z_i \sim \delta e^{-\delta Z}$

δ = up-to-derated failure rate
e = shape parameter of gamma prior for up-to-derated failure rate δ
Δ = inverse scale parameter of gamma prior for up-to-derated failure rate δ

Now let the up-to-derated failure rate δ have a gamma prior distribution:

$$\theta_3(\delta) = \frac{\Delta^e}{\Gamma(e)} \delta^{e-1} \exp(-\delta\Delta), \ \delta > 0 \tag{16}$$

Similarly as above, skipping two intermediate steps, the conditional posterior density of δ:

$$h_3(\delta | \tilde{z}) = \frac{1}{\Gamma(o+e)} (z_T + \Delta)\delta^{o+e-1} \exp[-\delta(z_T + \Delta)], \tag{17}$$

which is also a Gamma [o+e, $(z_T + \Delta)^{-1}$].

2.2.4 Derated to Up Recovery Transition Rate (β), e.g. from u1 to x2, or u2 to z2 in Fig. 2.2.2

Let, *k*= number of occurrences of up times before going derated

$u_T = \sum_{1}^{o} U_i$ total derated times before going up for *k* # of such occurrences

$U_i \sim \beta e^{-\beta U}$

β = derated-to-up recovery rate

ϕ = shape parameter of gamma prior for derated-to-up recovery rate β

f = inverse scale parameter of gamma prior for derated-to-up recovery rate

Now let the derated-to-up failure rate β have a gamma prior distribution:

$$\theta_4(\beta) = \frac{\phi^f}{\Gamma(f)}\beta^{f-1}\exp(-\beta\phi),\ \beta > 0 \tag{18}$$

Similarly as above, skipping two intermediate steps, the conditional posterior density of β:

$$h_4(\beta \mid \tilde{u}) = \frac{1}{\Gamma(k+f)}(u_T + \phi)\beta^{k+f-1}\exp[-\beta(u_T + \phi)], \tag{19}$$

which is also a Gamma [k+f, $(u_T + \phi)^{-1}$].

2.2.5 Derated to Down Failure Transition Rate (α); e.g. from s1 to y2 in Fig. 2.2.2

Let, j= number of occurrences of derated times before going down

$s_T = \sum_1^j S_i$ total derated times before going down for j # of such occurrences

$S_i \sim \alpha e^{-\alpha s}$

α = derated-to-down failure rate

g= shape parameter of gamma prior for derated-to-down failure rate α

ψ = inverse scale parameter of gamma prior for derated-to-down failure rate rate α

Now let the derated-to-down failure rate α have a gamma prior distribution:

$$\theta_5(\alpha) = \frac{\psi^g}{\Gamma(g)}\alpha^{g-1}\exp(-\alpha\psi),\ \alpha > 0 \tag{20}$$

Similarly as above, skipping two intermediate steps, the conditional posterior density of α:

$$h_5(\alpha \mid \tilde{s}) = \frac{1}{\Gamma(j+g)}(s_T + \psi)\alpha^{j+g-1}\exp[-\alpha(s_T + \psi)], \tag{21}$$

which is also a Gamma [j+g, $(s_T + \psi)^{-1}$].

2.2.6 Down to Derated Recovery Transition Rate (γ), e.g. from w1 to s1, or w2 to u2 in Fig. 2.2.2

Let, p=number of occurrences of down times before going derated

$w_T = \sum_1^p W_i$ total down times before going derated for p # such occurrences

$W_i \sim \gamma e^{-\gamma w}$

γ = down-to-derated recovery rate

h = shape parameter of gamma prior for down-to-derated recovery rate γ

π = inverse scale parameter of gamma prior for down-to-derated recovery rate γ

Now let the down-to-derated recovery rate γ have a gamma prior distribution:

$$\theta_6(\gamma) = \frac{\pi^h}{\Gamma(h)}\gamma^{h-1}\exp(-\gamma\pi),\ \gamma > 0 \tag{22}$$

Similarly as above, skipping two intermediate steps, the conditional posterior density of γ:

$$h_6(\gamma \mid \widetilde{w}) = \frac{1}{\Gamma(p+h)} (w_T + \pi)\gamma^{p+h-1} \exp[-\gamma(w_T + \pi)], \tag{23}$$

which is also a Gamma [p+h, $(w_T + \pi)^{-1}$].

3. SIMULATIONS FOR THREE-STATE UNIT TO FIND THE P.D.F. OF UP, DOWN, DERATED

Given the following example covering the first n=5 counts or episodes of each of six different sojourn times as in Fig. 2.2.2 inspired from Fig. 2.2.1; n=100,000 simulations for various derivations 2.2.1 - 2.2.6 are conducted. Given the input table as in Fig.3.1 and 3.2, the Java program will compute the statistical measures of three random variables. Click ERBDC"s three-state SL at www.areslimited.com for Java applications. Probability distributions of equations (5)-(7) are approximated: f(UP)=Normal(0.267, 0.107); f(DER)=Normal(0.433, 0.100), f(DOWN)= Normal(0.299, 0.106). The limits in Fig. 3.1 are: {UP_u = .12, UP_L = .46}, { DER_u = .27, DER_L = .60}, { $DOWN_u$ = .14, $DOWN_L$ = .49}. Medians , and first(Q_1) and third (Q_3) quartiles are computed in Fig. 3.2. {UP_{Q1} =.18, UP_{Q3} =.33}, { DER_{Q1} = .36, DER_{Q3} = .50}, { $DOWN_{Q1}$ = .22, $DOWN_{Q3}$ = .37}. Medians (M) =~ Means (E) are almost identical due to quasi-symmetricity.

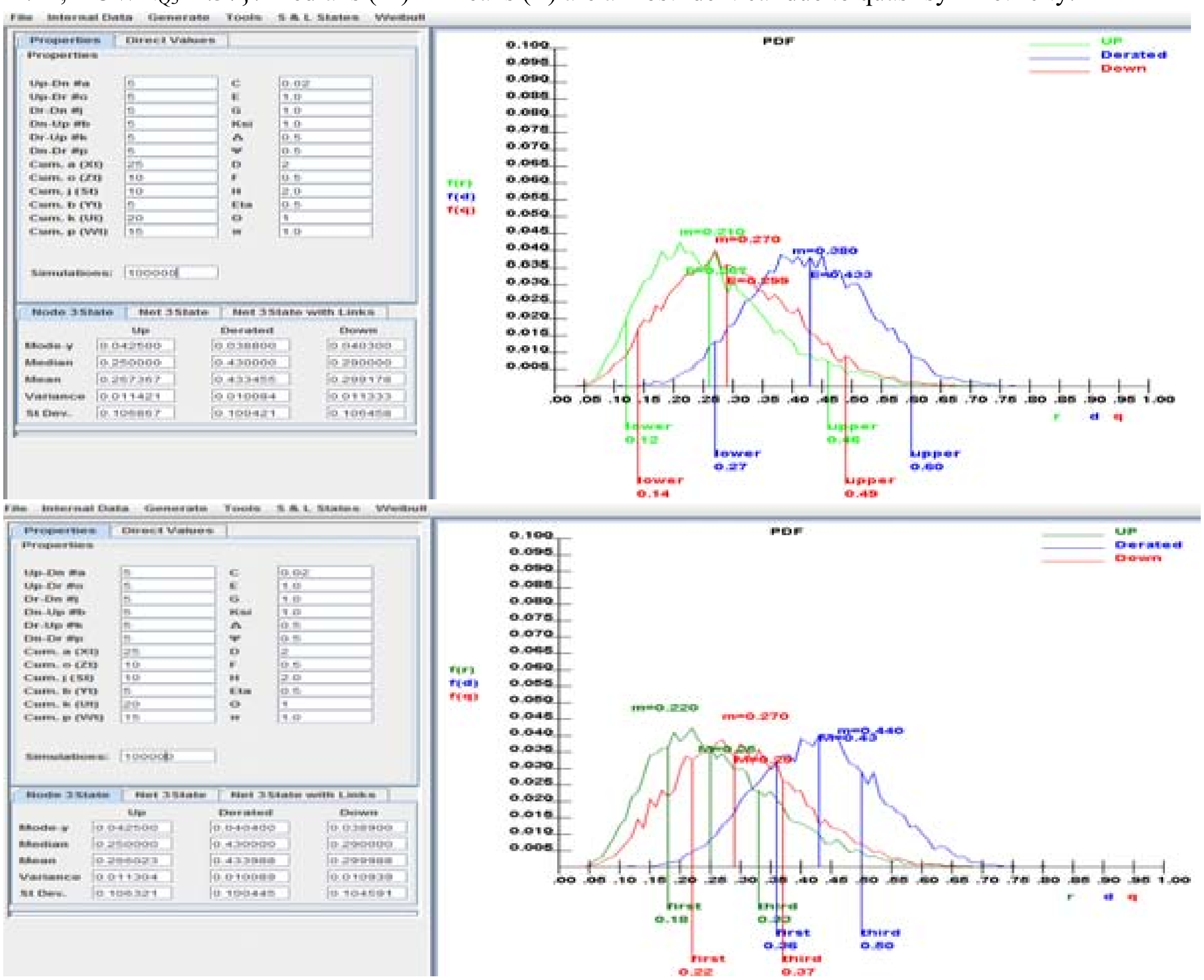

Figure 3.1 and Fig. 3.2: Given the input table, the p.d.f. of the 3 states are plotted for up (r), derated (d), and down (q) for a 90% confidence level showing mode (m), mean (E) with upper & lower confidence for n=100,000 simulation runs. Fig 3.2 is same as Fig. 3.1 but with Median (M), first and third quartiles.

4. DISCUSSIONS AND CONCLUSIONS

In this research we have studied the basic theory and application of the Sahinoglu-Libby (SL) p.d.f. both for a conventional two-state (up and down) and additionally three-state (up, derated and down) diagram applicable to hardware units and networks (Sahinoglu, 2007; Sahinoglu *et al*., 2005; and Billinton, 1970) in practical engineering projects. The authors wish to study an important pillar of trustworthy computing, namely the availability of a system when the underlying units have two or three states with derated included, instead of the usual easier two-state assumption. In the theoretical section of the references (Sahinoglu, 2007 and Sahinoglu *et al*., 2005), a detailed analytical derivation of the univariate SL(α, β, L) p.d.f. as originally noted in Sahinoglu"s Ph.D. dissertation (Sahinoglu, 1981) is presented with reference to an empirical Bayesian process for informative priors using squared-error and other loss functions.

Two-state univariate SL can be analytically derived in a closed form solution for the target unavailability expression of $\lambda/(\lambda+\mu)$, but the three-state multivariate analysis must employ Monte Carlo simulations to arrive at an approximately normal distributional result. It is practically infeasible to find closed form solutions for the random variables of UP, DERATED and DOWN expressed by equations 5 to 7 due to a multiplicity of products and summations of Gamma p.d.f.s expressed in the denominator term of Section 2.2 $\mu\beta+\mu\gamma+\alpha\beta+\lambda\beta+\lambda\gamma+\delta\mu+\delta\alpha+\lambda\mu+\delta\gamma$. At the final analysis as shown in Section 3, the resulting distributions for the three parameters, UP, DERATED and DOWN are approximated by normal distributions owing to normal probability plots (one way of goodness-of-fit tests) conducted favorably. The distributions in Section 3 are quasi-symmetrical with E (Mean) and M (median) almost equal, although slightly right-skewed because Mean>Median>Mode.

Therefore, SL(α, β, L) p.d.f. is the continuous probability density function of the random variable of unavailability (or availability when reparametrized) of a two-state unit. For those units whose life time can be decomposed into operating (UP), derated (DER), and non-operating (DOWN) states in a tripod setting. up, derated and down times are assumed to be distributed with respect to the general gamma models where both shape and scale parameters are different from each other. The resultants p.d.f.s are nearly normal.

Further, analytical difficulties in calculating the closed-form moments of the said random variable of unavailability (or availability) are outlined, suggesting empirical Bayesian estimators using informative priors with respect to varying forms of squared-error loss functions. Due to infeasibility of closed form analytical solutions for a three-state version, Monte Carlo simulation technique is rightfully selected as a mathematically tractable and compatible model to calculate the UP and DER and DOWN probabilities for a three-state hardware repairable unit. The two-state SL"s closed form was derived (Sahinoglu, 2007 and Sahinoglu *et al*., 2005). Due to infeasibility of closed-form solutions of the three-state SL model, the analyses have been conducted by Monte Carlo simulations using the empirical Bayesian principles so as to estimate the full and derated availability of a repairable hardware unit, or ingress-egress (input-output) network composed of such three-state units.

Given the apriori parameters with respect to a squared error loss function, we first obtain the informative mean estimator. On the other hand, the median is the summary measure when absolute error loss is assumed. Mode is the maximum likelihood value. These summary measures are all shown in the plots of the Java applications in Section 5, also observable in the ERBDC section of the educational website www.areslimited.com (Sahinoglu, 2007 and Sahinoglu *et al*., 2005). Network applications for large and very large networks are studied using Monte Carlo simulations in reference (Sahinoglu et al., 2010).The large number of simulation runs stretching to 100,000 assure that the simulation solutions are converging to the true result by way of convergence. Another question that is of interest to simulation experts is: Why do we use Gamma priors? Primary reasons are as follow:

1) Gamma priors are mathematically tractable, i.e., when Gamma is used as a prior, you get the posterior density also a Gamma. That is we take advantage of the statistical conjugacy property of gamma when negative exponential density is used as time to failure distribution for the counting Poisson process.
2) In the 1980s, fast computing for Monte Carlo Simulation was not available, let alone pervasive as today. However now you can work with any prior you choose, and you can achieve the posterior without a computational burden. However the disadvantage is that you lose on the exactitude; that is Monte Carlo is only approximating the true solution, hoping it will converge as you reach millions of simulation runs.
3) As important as the statistical-conjugacy property and mathematical tractability; if you possess real data, GOFT (Goodness of Fit Tests) can be applied to see which fit goes better? Now, what you do is perform goodness of fit test on the prior data you have (empirical). Or if you are not an Empirical Bayesian, and otherwise, you directly may assume the Gamma p.d.f. as your prior, based on popular judgment such as what you know from the past experience regarding the unknown lambda (failure rate) or mu(repair rate). In past studies, collected empirical data resulted in Gamma priors (Sahinoglu, 1981).
4) Gamma p.d.f. is one of most versatile and has a large domain of applicability from hyper- or hypo-exponentiality to negative exponentiality and even rectangular p.d.f. when the appropriate shape and scale parameters are employed. Again, with the subsequent field data dominating over the prior parameters, the effect of priors will diminish unless you are dealing with a relatively few sample sizes. In that case, the prior judgment is important to influence when the samples are overly destructive (like collecting data on space programs waiting for the satellite crash data to happen!), and may need a life time to pass like in death for cancer patients. However in our case of millions of runs, the priors will lose its dominance as field data size approaches large. After the analyses, the approximate closed form p.d.f.s will be derived as shown in Section 3.1 owing to the favorable results by normal probability plots as one way of GOFT. For further analysis, GOFT will be conducted to fit the p.d.f. in Fig. 3.1, 3.2 to those of Sahinoglu-Libby.

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