Spatial Hierarchical Models for Extremes: Modeling Both Climate and Weather Effects

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Abstract

This paper and talk will discuss the applied aspects of joint work with Mathieu Ribatet, Department of Mathematics, Université Montpellier II, and Anthony Davison, Institute of Mathematics, École Polytechnique Fédérale de Lausanne. The complete paper Ribatet et al. [2010] is under review.

Weather data are characterized by two types of spatial effects: climate effects that occur on a regional scale and weather effects that occur on a local scale. In terms of a statistical model, one can view climate effects as how the marginal distribution varies by location and the weather effects as characterizing the joint behavior. We extend recent work in spatial hierarchical models for extremes by employing a max-stable random process at the data level of the hierarchy, thereby accounting for the weather spatial effects which had often been ignored. Because the known max-stable process models can be written in closed form only for the bivariate case, we employ composite likelihood methods to implement them in our hierarchical model. Appropriate uncertainty estimates are obtained via an information sandwich approach.

1 Motivation: Climate and Weather Effects in Extreme Data

Geophysical data are spatial, thus to model extreme geophysical data, one must account for its spatial behavior. This is difficult, as the data arise from a combination of both climate and weather effects. For discussion purposes, we think of climate as the distribution from which weather events are drawn. Since climate varies with location, climate spatial effects are how the marginal behavior of extremes vary over a study region. Weather spatial effects are how a particular weather event affects multiple locations.

Figure 1 illustrates these two ideas. The data used to produce both figures come from a 20-year run of a regional climate model (RCM) for the western part of North America [Leung et al., 2004]. The left panel of Figure 1 shows the estimated 20-year precipitation return level, a summary measure of climatological precipitation extremes. This panel shows how the 20-year return level for summer (June, July, August) precipitation varies over the study region [Cooley and Sain, 2010]. The right panel of Figure 1 shows the spatial extent of a particular extreme precipitation weather events. Specifically, this panel shows locations whose annual maximum precipitation occur on the same day, with each color corresponding to a particular day. Clearly, a storm affects more than one location.

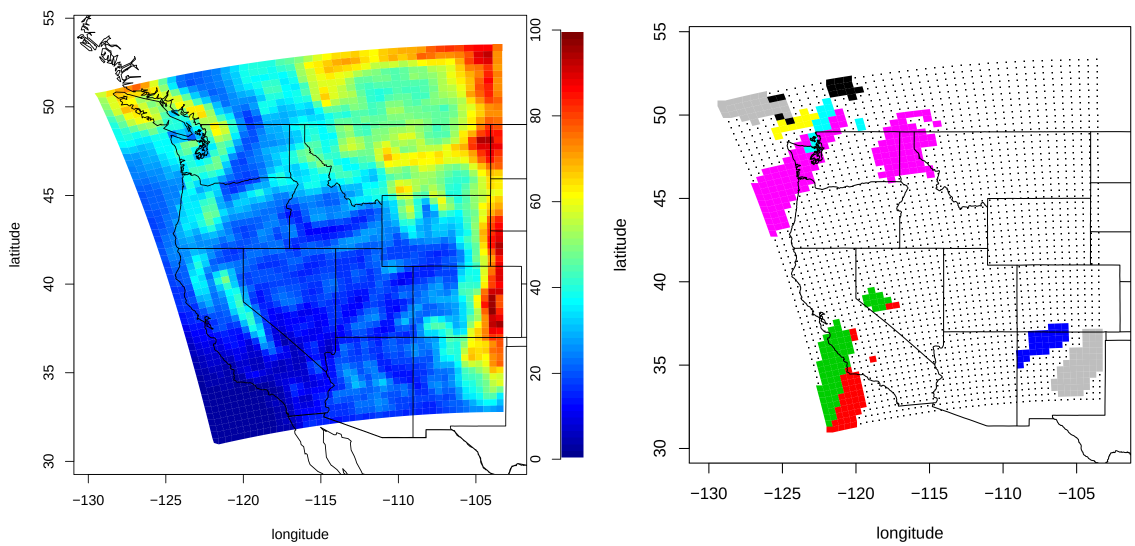

Figure 1: The left panel depicts "climatological" extremes; it shows the estimated 20-year precipitation return level for the RCM control run data. The right panel depicts "weather" extremes, each color shows locations which have their annual maximum observation occur on the same day.

Spatial extremes studies have typically focused on modeling either the climate or weather effects of extreme phenomena, and until recently have not tried to capture both spatial effects. Climate studies of extremes aim to characterize how the behavior of extremes changes over a study region, and these studies typically ignore any spatial effects due to weather events and make some sort of independence assumption for the data. For example, when constructing Bayesian hierarchical models for climatological extremes Cooley and Sain [2010]; Sang and Gelfand [2010], and Cooley et al. [2007] assume that the data at each location, given the parameters that describe the marginal distribution at that location, are conditionally independent from the data at all other locations. The right panel of Figure 1 shows that this assumption is clearly false, as the storm event is not limited to a single cell. On the other hand, weather effects for extreme data can be captured by max-stable process models [Smith, 1990; Schlather, 2002; Kabluchko et al., 2009], but such models generally implicitly assume a common marginal distribution for all locations. It can be difficult to apply max-stable process models to real data and capture the complexity of how the marginal distributions vary over a study region.

2 Max-stable Process Models

Let $Y_m(x)$, $x \in \mathcal{D}$, $m \geq 1$ be i.i.d replications of a stochastic process. Asymptotic theory for extremes tells us that, provided the limit exists and is non-degenerate, $\max_{m=1,\dots n} a_n(x)^{-1}\{Y_m(x) - b_n(x)\}$ converges weakly to a max-stable process [de Haan, 1984]. Given observations that arise as block (e.g annual) maxima, it is natural to assume that they should approximately follow a max-stable process.

Max-stable processes can be developed from a Poisson process perspective, and from this point-of-view several models have been suggested [Smith, 1990; Schlather, 2002; de Haan and Pereira, 2006; Kabluchko et al., 2009]. Although all are valid stochastic processes, only the bivariate distribution is known in closed form for each. Additionally, in their basic form, all of the models assume a common marginal distribution, which, in terms of applications, implies that any regional effects have already been accounted for. Although the general methodology we propose could use any of the max-stable process models, in this work we focus on the Gaussian extreme value process model proposed by Smith [1990]

$$Z(x) = \max_{k \geq 1} \zeta_k \varphi(x - s_k) \tag{1}$$

where $\{(\zeta_k, s_k)\}_{k\geq 1}$ are the points of a Poisson process on $\mathbb{R}_*^+ \times \mathcal{D}$, $\mathcal{D} \subset \mathbb{R}^d$ with intensity $\mathrm{d}\Lambda(\zeta, s) = \zeta^{-2}\mathrm{d}\zeta\mathrm{ds}$ and φ is the zero mean d-variate Normal density with covariance matrix Σ. As formulated, $Z(x)$ has unit Fréchet margins and its bivariate cdf is given by

$$\Pr[Z(x_i) \leq z_1, Z(x_j) \leq z_2] = \exp\left\{-\frac{1}{z_1}\Phi\left(\frac{a}{2} + \frac{1}{a}\log\frac{z_2}{z_1}\right) - \frac{1}{z_2}\Phi\left(\frac{a}{2} + \frac{1}{a}\log\frac{z_1}{z_2}\right)\right\}, \tag{2}$$

where $a^2 = (x_i - x_j)^T\Sigma^{-1}(x_i - x_j)$ and Φ denotes the cdf of the univariate standard normal distribution. If $Y(x_i)$ is not Fréchet distributed but instead has a generalized extreme value (GEV) distribution with parameters $\{\mu(x_i), \sigma(x_i), \xi(x_i)\}$, then by applying the transformation

$$t : Y(x_i) \mapsto \left\{1 + \xi(x_i)\left(\frac{Y(x_i) - \mu(x_i)}{\sigma(x_i)}\right)\right\}^{\xi(x_i)},$$

the model can be formulated in terms of the (not necessarily identical) GEV marginals.

Since only the bivariate distributions are known, an obvious approach to fitting max-stable models is to employ a pairwise likelihood. Padoan et al. [2010] use the pairwise likelihood approach on a study of annual maximum precipitation for a region of the southeast United States. To account for non-stationarity in the marginal distributions, the analysis assumes the GEV parameters follow a trend surface that is a function of location and available covariates.

Often however, the regional effects are such that the available covariates fail to sufficiently explain the variation of the marginal distribution over the study location. As mentioned earlier, one approach to capture the regional effects has been to construct a hierarchical model where the marginal parameters of the extreme value distribution follow a stochastic process over the study region. The difficulty with such models lies in accounting for dependence in the data at the local scale. Cooley et al. [2007], Sang and Gelfand [2009] and Cooley and Sain [2010] all disregard local dependence and assume that the data at each location, conditioned on the marginal parameters, are independent of the data at other locations. An alternative approach proposed by Sang and Gelfand [2010] and Fuentes et al. [2009] is to use a copula to model the local dependence. While the copula approaches do account for local dependence, both the Gaussian copula of Sang and the stick-breaking copula approach of Fuentes are asymptotically independent [Sibuya, 1960], and thus it is questionable whether these models are adequate for describing dependence at extreme levels.

Our approach is to use a max-stable process model within a hierarchical framework; the max-stable model provides a theoretically justified model for the local dependence, and the hierarchy allows for flexibility in modeling how the regional effects vary the marginal behavior. Inference for composite likelihoods is well developed in the frequentist paradigm, but is only beginning to be explored in the Bayesian setting. Ribatet et al. [2010] build on frequentist work done to develop likelihood ratio tests to propose adjustments to the likelihood which result in posterior distributions which have appropriate inference properties.

3 Spatial Hierarchical Model for Precipitation Extremes

The model we employ has the standard data-process-prior framework of most hierarchical models. We seek to make inference on both the data level parameters $\boldsymbol{\theta}_1$ and process level parameters $\boldsymbol{\theta}_2$. Our model is structured as:

$$\pi(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2|\mathbf{y}) \propto \underbrace{\pi(\mathbf{y}|\boldsymbol{\theta}_1)}_{data}\underbrace{\pi(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2)}_{process}\underbrace{\pi(\boldsymbol{\theta}_2)}_{prior}, \tag{3}$$

where $\mathbf{y}$ denotes our data. Spatial modeling of local effects takes place at the data level, via the max-stable process model, and spatial modeling of the regional effects occurs at the process level.

It is at the data level where our model differs from most previous hierarchical models for extremes as we employ Smith's (1990) Gaussian extreme value process. Assume we have locations $x_1, \ldots, x_K$ in a study region $\mathcal{D}$. Let $y_m(x_i)$ be the maximum measurement for location i and block (e.g., year) $m = 1, \ldots, n$ and let $\mathbf{y}_m = (y_m(x_i), \ldots, y_m(x_K))$. Since the full likelihood for K locations is unknown, we set

$$\pi(\mathbf{y}|\boldsymbol{\theta}_1) = \prod_{m=1}^{n} \prod_{i=1}^{K-1} \prod_{j=i+1}^{K} f\{t(y_m(x_i)), t(y_m(x_j)); \Sigma\} |J(y_m(x_i))||J(y_m(x_j))|, \tag{4}$$

where f is the density arising from (2), and $|J(\cdot)|$ is the jacobian of the mapping t. By using the pairwise likelihood in place of the full likelihood, we recognize that (3) will not be a standard posterior and we discuss the necessary adjustment for inference in Ribatet et al. [2010].

In the process level, we make the assumption, common in spatial hierarchical modeling, that the parameters that define the marginal distribution are spatially varying. In this work, we assume that the GEV parameters follow independent Gaussian processes: $\mu(x)$ has mean $\mathbf{X}(x)^T \boldsymbol{\beta}_\mu$ where $\mathbf{X}(x)$ is the vector of covariates at location x and $\mathrm{Cov}\{\mu(x_i), \mu(x_j)\} = \tau_\mu \exp(-\|x_i - x_j\|/\omega_\mu)$. The processes $\sigma(x)$ and $\xi(x)$ are analogously defined. We have used a relatively simple process model as our primary focus is on the behavior of the hierarchical model with the pairwise likelihood, but more complex process modeling is certainly possible. Both Sang and Gelfand [2009] and Cooley and Sain [2010] use a multivariate specification in the process level of their hierarchical extremes models. In the process level we also need to specify a prior on the covariance matrix Σ which appears in the Smith model and we use an independent Wishart prior with fixed degrees of freedom ν and parameter matrix V. Hence, letting $\boldsymbol{\mu} = \{\mu(x_1), \ldots, \mu(x_K)\}^T$ and defining $\boldsymbol{\sigma}$ and $\boldsymbol{\xi}$ analogously, the process level of our model is given by

$$\pi(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2) = \varphi(\boldsymbol{\mu}|\beta_\mu, \tau_\mu, \omega_\mu)\varphi(\boldsymbol{\sigma}|\beta_\sigma, \tau_\sigma, \omega_\sigma)\varphi(\boldsymbol{\xi}|\beta_\xi, \tau_\xi, \omega_\xi) f_\Sigma(\Sigma|\nu, V) \tag{5}$$

where φ denotes the multivariate density that arises from observing the Gaussian process at locations $x_i, i = 1, \ldots, K$ and f_Σ is the Wishart density.

The prior level assumes independent priors on all parameters introduced at the process level. Conjugate normal priors are assumed for all regression parameters $\beta_\cdot$, conjugate inverse gamma priors are assumed for $\tau_\cdot$, and gamma priors are assumed for the range parameters $\omega_\cdot$. In all cases, the variance is set to be large so that the priors are relatively non-informative.

In a simulation study, our results show that our model adequately captures both the marginal effects as well as spatial dependence in the data. We apply the model to precipitation data in Switzerland.

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