MONITORING LOCATION: A NONPARAMETRIC CONTROL CHART BASED ON THE SIGNED-RANK STATISTIC

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ABSTRACT

Standard control charts are often based on the assumption that the observations follow a specific parametric distribution, such as the normal. In many applications we do not have enough information to make this assumption and in such situations, development and application of control charts that do not depend on a particular distributional assumption is desirable. Nonparametric or distribution-free control charts can serve this wider purpose. A nonparametric exponentially weighted moving average (NPEWMA) control chart combines the advantages of a nonparametric control chart with the better shift detection properties of a traditional EWMA chart. A NPEWMA chart for the median of a symmetric continuous distribution was introduced by Amin and Searcy (1991) using the Wilcoxon signed-rank statistic (see Gibbons and Chakraborti, 2003). This is called the nonparametric exponentially weighted moving average Signed-Rank (NPEWMA-SR) chart. However, important questions remained unanswered regarding the practical implementation as well as the performance of this chart. In this paper we address these issues with a more in-depth study of the two-sided NPEWMA-SR chart. A Markov chain approach is used to determine the run-length distribution and the associated performance characteristics. Detailed guidelines for selecting the design parameters are provided for practical implementation along with an illustrative example.

1. INTRODUCTION

A control chart is a statistical scheme (typically a two dimensional graphic) devised for the purpose of monitoring the statistical stability of a process. An efficient control chart must continue sampling as long as the process is in-control and must give a signal to stop sampling as quickly as possible when the process becomes out-of-control. Shewhart control charts are the most popular in practice because of their simplicity, ease of application, and the fact that these versatile charts are quite efficient in detecting moderate to large shifts. However, the Shewhart chart is not as effective as the exponentially weighted moving average (EWMA) chart for detecting relatively small shifts (see e.g. Montgomery, 2009 pages 400 and 419). The superiority of the EWMA chart stems from the fact that it uses information in the data from start-up and not the most recent time point only. The existing literature on the EWMA chart is quite substantial and continues to grow (see e.g. the overview on EWMA charts by Ruggeri et al. (2007)). In typical applications of the EWMA chart it is assumed that the underlying process distribution is normal (or, at least, approximately so). However, if normality is in doubt a nonparametric (NP) chart is more desirable, since then the performance of the traditional EWMA chart can get seriously degraded.

2. THE NPEWMA-SR CONTROL CHART

A nonparametric control chart can be applied for data from all continuous distributions and it seems natural to consider nonparametric analogs of the traditional EWMA chart. Although Amin and Searcy (1991) proposed a nonparametric EWMA (NPEWMA) control chart based on the signed-rank statistic (we label this the NPEWMA-SR chart), much work remained to be done. Chakraborti and Graham (2007) noted that "…more work is necessary on the practical implementation of the (NPEWMA-SR) chart…". In this paper we perform an in-depth study of the NPEWMA-SR chart.

Suppose that X_{ij}, $i = 1,2,3, \ldots$ and $j = 1,2, \ldots, n$, denote the j^{th} observation in the i^{th} rational subgroup of size $n > 1$. Let R_{ij}^{+} denote the rank of the absolute values of the differences $|X_{ij} - \theta_0|$, $j = 1,2, \ldots, n$, within the i^{th} subgroup. Define $SR_i = \sum_{j=1}^{n} sign(X_{ij} - \theta_0) R_{ij}^{+}$, $i = 1,2,3, \ldots$, where θ_0 is the known or the specified or the target value of the median, θ, that is monitored. Note that the statistic SR is linearly related to the better-known Wilcoxon signed-rank statistic T_n^{+} through the relationship $SR = 2T_n^{+} - n(n+1)/2$ (the reader is referred to Gibbons and Chakraborti (2003) page 197 for more details on the T_n^{+} statistic). Assuming, without any loss of generality $\theta_0 = 0$, the plotting statistic for the NPEWMA-SR chart is $Z_i = \lambda\, SR_i + (1-\lambda) Z_{i-1}$ for $i = 1,2,3, \ldots$, where $0 < \lambda \le 1$ is a smoothing constant and the starting value is taken as $Z_0 = 0$. The upper and lower control limits of the NPEWMA-SR chart are given by $= \pm L \sqrt{\left(\frac{n(n+1)(2n+1)}{6}\right)\left(\frac{\lambda}{2-\lambda}\right)(1-(1-\lambda)^{2i})}$ and $CL = 0$. When the chart has been running for several time periods we can use the steady-state control limits given by $= \pm L \sqrt{\left(\frac{n(n+1)(2n+1)}{6}\right)\left(\frac{\lambda}{2-\lambda}\right)}$.

3. THE RUN-LENGTH DISTRIBUTION AND IMPLEMENTATION OF THE CHART

In the developments that follow:

i. We study two-sided charts; the methodology can easily be modified where a one-sided chart is more meaningful.
ii. We use the steady-state control limits; this significantly simplifies the calculation of the run-length distribution via the Markov chain approach.
iii. We examine the average run-length (*ARL*) as a performance measure and, for a more thorough assessment of the chart's performance, we also calculate and study the standard deviation (*SDRL*), the median (*MDRL*), the 1st and 3rd quartiles as well as the 5th and 95th percentiles for an overall assessment of the run-length distribution. It should be noted that Amin and Searcy (1991) only evaluated the *ARL*.

3.1. Computation of the run-length distribution

To design the NPEWMA-SR chart and study its performance, we evaluate their run-length distribution and the associated characteristics. We use mainly a Markov chain approach as this approach is easy to program and it offers a unified / flexible method to evaluate the performance measures. The details are omitted here to conserve space (the reader is referred to Graham et al. (2009) and (2011) for more details on the Markov chain approach). In this paper, we only give the results relating to the (i) in-control (IC) robustness (since the NPEWMA-SR chart is nonparametric, the IC run-length characteristics should remain the same for all continuous distributions) and (ii) out-of-control (OOC) performance comparisons (the in-control *ARL* (denoted ARL_0) is fixed at an acceptably high value in order to compare the charts; the chart with the smallest or lowest out-of-control *ARL* (denoted ARL_δ) for a change or shift is then selected to be the winner).

3.2. Choice of the design parameters

The choice of the design parameters (λ, L) entails two steps: First, one has to (use a search algorithm to) find the (λ, L) combinations that yield the desired ARL_0. Second, one has to choose, among these (λ, L) combinations, the one that provides the best performance i.e. the smallest ARL_δ for the shift (δ) that is to be detected. Note that, the smoothing parameter $0 < \lambda \leq 1$ is typically selected first (which depends on the magnitude of the shift to be detected) and then the constant $L > 0$ is selected (which determines the width of the control limits i.e. the larger the value of L, the wider the control limits and vice versa).

3.3. Implementation of the NPEWMA-SR chart

To implement the chart, a practitioner needs values of the design parameters (λ, L). The first step is to choose λ. If small shifts (roughly 0.5 standard deviations or less) are of primary concern the typical recommendation is to choose a small λ such as 0.05, if moderate shifts (roughly between 0.5 and 1.5 standard deviations) are of greater concern choose λ = 0.10, whereas if larger shifts (roughly 1.5 standard deviations or more) are of concern choose λ = 0.20 (see e.g. Montgomery (2009), page 423). Next we choose L, in conjunction with the chosen λ, so that a desired nominal ARL_0 is attained. Table 1 lists some (λ, L)-combinations for the popular ARL_0 values of 370 and 500 and for subgroups of size n = 5 and n = 10 for the NPEWMA-SR chart. These tables should be very useful for implementing the NPEWMA-SR chart in practice.

Table 1. (λ, L)-combinations for the NPEWMA-SR chart for nominal ARL_0 = 370 and 500.[1]

	Nominal ARL_0 = 370		**Nominal ARL_0 = 500**	
Shift to be detected	**(λ, L)**	**Attained ARL_0**	**(λ, L)**	**Attained ARL_0**
	***n* = 5**			
Small	(0.05, 2.481)	370.29	(0.05, 2.602)	499.83
Moderate	(0.10, 2.668)	370.13	(0.10, 2.775)	500.11
Large	(0.20, 2.764)	369.91	(0.20, 2.852)	499.27
	***n* = 10**			
Small	(0.05, 2.486)	370.49	(0.05, 2.610)	500.67
Moderate	(0.10, 2.684)	370.09	(0.10, 2.794)	500.13
Large	(0.20, 2.810)	370.19	(0.20, 2.905)	498.92

[1]Table 1 is more extensive and unlike in Amin and Searcy (1991) who give some (λ, *UCL*)-values.

4. PERFORMANCE COMPARISON

We compare the NPEWMA-SR to the traditional EWMA chart for subgroup averages (we label this the EWMA-$\bar{X}$ chart), the NPEWMA chart based on signs which was proposed by Graham et al. (2009) (we label this the NPEWMA-SN chart) and the runs-rules enhanced Shewhart-type SR charts i.e. the basic *1-of-1* chart, the *2-of-2* DR chart and the *2-of-2* KL Shewhart-type SR charts (see Chakraborti and Eryilmaz (2007) for a detailed description of the *2-of-2* DR and KL charts, respectively). Our study includes a wide collection of symmetric distributions including the normal and non-normal distributions: (a) the standard normal distribution, $N(0,1)$; (b) the scaled Student's t-distribution, $t(v)/\sqrt{\frac{v}{v-2}}$, with degrees of freedom v = 4 and 8, respectively; (c) the Laplace (or double exponential) distribution, $DE(0,1/\sqrt{2})$; (d) the logistic distribution, $LG(0,\sqrt{3}/\pi)$; (e) the contaminated normal (CN) distribution: a mixture of $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$, represented by $(1 -$

$\alpha)N(0,\sigma_1^2) + \alpha N(0,\sigma_2^2)$. Note that all distributions in the study have mean/median 0 and are scaled such that they have a standard deviation of 1 so that the results are easily comparable across distributions. For the CN distribution the σ_i's are chosen so that the standard deviation of the mixture distribution equals 1, that is, $(1-\alpha)\sigma_1^2 + \alpha\sigma_2^2 = 1$. We take $\sigma_1/\sigma_2 = 2$ and the level of contamination $\alpha = 0.05$.

We first compare the EWMA-type charts, i.e. the NPEWMA-SR chart to the traditional EWMA-$\overline{X}$ and the NPEWMA-SN charts, respectively. Following this, we compare the NPEWMA-SR chart to the *1-of-1*, the *2-of-2* DR and the *2-of-2* KL Shewhart-type SR charts.

4.1. In-control robustness

A Markov chain approach was used in the calculations for the two NPEWMA charts whereas for the traditional EWMA-$\overline{X}$ chart, the values of the IC run-length characteristics were estimated using 100,000 simulations as the exact closed-form expressions for the run-length distribution is not available for all the distributions considered in the study; the main stumbling block being the exact distribution of the mean (i.e $\overline{X}$) for small subgroup sizes. The results are shown in Table 2 for λ = 0.05, 0.10 and 0.20, respectively. Note that, the values of L were chosen such that in each case $ARL_0 \approx 500$ and, in case of the EWMA-$\overline{X}$ chart, the values of L were chosen such that the $ARL_0 \approx 500$ for the N(0,1) distribution. The first row of each cell in Table 2 shows the ARL_0 and $SDRL_0$ values, respectively, whereas the second row shows the values of the 5th, 25th, 50th, 75th and 95th percentiles (in this order).

Table 2. Performance characteristics of the IC run-length distribution for the NPEWMA-SN chart, the NPEWMA-SR chart and the EWMA-$\overline{X}$ chart for selected (λ, L)-combinations and n = 10.

NPEWMA-SN chart			
(λ, L)	**(0.05, 2.612)**	**(0.10, 2.797)**	**(0.20, 2.933)**
All continuous distributions	501.04 (486.58) 39, 155, 352, 689, 1472	500.25 (491.88) 34, 150, 349, 690, 1482	499.64 (495.00) 30, 147, 348, 691, 1488
NPEWMA-SR chart			
(λ, L)	**(0.05, 2.610)**	**(0.10, 2.794)**	**(0.20, 2.905)**
All symmetric continuous distributions	500.67 (486.10) 40, 154, 352, 688, 1471	500.13 (491.61) 34, 150, 349, 690, 1481	498.92 (494.15) 30, 147, 347, 690, 1485
EWMA-$\overline{X}$ chart			
Dist | (λ, L)	**(0.05, 2.613)**	**(0.10, 2.815)**	**(0.20, 2.962)**
***N*(0,1)**	496.37 (482.62) 39, 152, 350, 681, 1462	498.96 (490.01) 34, 149, 349, 689, 1475	497.31 (492.20) 30, 147, 346, 688, 1479
***t*(4)**	480.84 (470.36) 38, 148, 337, 661, 1421	441.57 (436.35) 29, 131, 308, 608, 1309	367.65 (365.04) 22, 108, 255, 509, 1094
***t*(8)**	494.13 (478.31) 39, 153, 349, 682, 1445	490.80 (479.81) 33, 147, 344, 678, 1445	471.10 (466.43) 28, 137, 329, 653, 1407
Laplace	491.87 (479.56) 39, 150, 345, 675, 1450	477.52 (473.51) 32, 142, 331, 657, 1423	438.70 (434.15) 26, 129, 305, 607, 1300
Logistic	491.81 (479.10) 39, 152, 345, 677, 1452	491.58 (485.19) 33, 147, 342, 676, 1462	473.63 (471.09) 28, 138, 328, 654, 1416
CN	494.67 (479.24) 39, 152, 349, 683, 1448	487.51 (477.50) 33, 148, 343, 671, 1438	476.14 (473.16) 29 ,140, 331, 662, 1411

Several interesting observations can be made from an examination of Table 2:

i. As expected, both NPEWMA charts are IC robust for all λ and for all distributions under consideration.

ii. The EWMA-$\overline{X}$ chart is not IC robust and its run-length distribution has a higher variance as seen from the interquartile ranges. Its IC characteristics vary (sometimes dramatically) as the underlying distribution changes. For example, focussing on the ARL_0 as a measure of location, for λ = 0.20 the ARL_0 of the EWMA-$\overline{X}$ chart varies from 497.31 (when the underlying distribution is N(0,1)) to 367.65 (when the underlying distribution is t(4)). In addition, for λ = 0.20, the ARL_0 values of the EWMA-$\overline{X}$ chart are much smaller than 500 for all distributions other than the normal. This is problematic as there will be many more false alarms than what is nominally expected.

iii. The EWMA$-\overline{X}$ chart appears to be less IC robust for larger values of λ, especially for the *CN* distribution. Thus, this chart may be problematic when outliers are likely to be present.

4.2. Out-of-control chart performance comparison

For the OOC chart performance comparison it is customary to ensure that the ARL_0 values of the competing charts are fixed at (or very close to) an acceptably high value, such as 500 in this case, and then compare their ARL_δ values, for specific values of the shift δ; the chart with the smaller ARL_δ value is generally preferred. Table 3 shows the OOC performance characteristics of the run-length distribution for various distributions and shifts of size δ = 0.5(0.5)2.5 in the mean/median, expressed in terms of the population standard deviation (which, in our case, equals one), for λ = 0.05 and n = 10.

< Insert Table 3 >

A summary of our observations from the OOC performance characteristics shown in Table 3 is as follows:

i. The NPEWMA-SR chart outperforms the NPEWMA-SN chart for all distributions under consideration except for the Laplace distribution, for which the performances of the charts are very similar (which is not surprising in view of the ARE values (see Gibbons and Chakraborti, page 508)). Both nonparametric charts perform significantly better than the EWMA-$\overline{X}$ chart for all distributions except the normal with (δ < 1.5) and even then the performances of the charts are very comparable. Similar conclusions can be drawn for λ = 0.10 and 0.20 where the run-length characteristics of the NPEWMA-SR chart tends to 3 and 2, respectively, as the shift increases.

ii. For larger shifts in location ($\delta \geq 1.5$), all the values of the run-length characteristics of the NPEWMA-SR chart become smaller and ultimately converge to 4 as the shift increases (due to a restriction given in Graham et al. (2011)) and those of the NPEWMA-SN chart also become smaller and ultimately converge to 3 as this shift increases (due to a similar type of restriction) and those of the EWMA-$\overline{X}$ can (and do) get smaller.

Next we compare the OOC performance of the NPEWMA-SR chart to that of the Shewhart-type SR charts. Table 14 of Chakraborti and Eryilmaz (2007) give the *ARL* values for n = 10 for the *1-of-1*, the *2-of-2* DR and the *2-of-2* KL Shewhart-type SR charts, respectively. Note that the control limits were chosen such that the $ARL_0 \approx 480$ for each chart.

Table 4. *ARL* values under the N(0,1) distribution when n = 10.

Shift	***1-of-1*** ***UCL/LCL*** **= $\pm$ 55**	***2-of-2*** **DR** ***UCL/LCL*** **= $\pm$ 39**	***2-of-2*** **KL** ***UCL/LCL*** **= $\pm$ 37**	**NPEWMA-SR** **(λ = 0.05, L = 2.595)** ***UCL/LCL*** **= $\pm$ 8.153**
0.0	$\pm$ 480.00	$\pm$ 480.00	$\pm$ 480.00	$\pm$ 480.00
0.2	208.76	147.19	113.17	22.25
0.4	66.93	30.37	22.52	9.56
0.6	25.22	9.60	7.51	6.43
0.8	10.72	4.49	3.89	5.11
1.0	5.64	2.90	2.66	4.44
1.2	3.37	2.31	2.22	4.11

From Table 4 we find that:

i. The NPEWMA-SR chart far outperforms all charts for shifts in location of 0.6 standard deviations or less.
ii. For shifts in the location of 0.8 standard deviations and larger, the performances of the charts are similar, particularly that of the runs-rule enhanced charts and the NPEWMA-SR charts.
iii. The *ARL* of the NPEWMA-SR charts tends to 4 as the shift increases. This is due to the restriction which is given in equation (10) of Graham et al (2011).

5. EXAMPLE

To illustrate the effectiveness and the application of the nonparametric charts when normality is in doubt we use some simulated data from a Logistic distribution with location parameter 0 and scale parameter $\sqrt{3}/\pi$: $LG(0, \sqrt{3}/\pi)$, so that the observations come from a symmetric distribution with a median of zero and a standard deviation of 1. Suppose that the median increases or has sustained an upward step shift of 0.5. Accordingly, subgroups each of size 5 (n = 5) were generated from the Logistic distribution with the same scale parameter but with the location parameter equal to 0.5, resulting in observations that have a median of 0.5 and a standard deviation of 1.

For the EWMA-$\bar{X}$, NPEWMA-SN and NPEWMA-SR and charts we set the chart design parameters (λ, L) = (0.05, 2.488), (0.05, 2.484) and (0.05, 2.481), respectively, so that $ARL_0 \approx 370$ for each chart. It should be noted that the industry standard ARL_0 value of 370 is far from being attainable when using the *1-of-1* Shewhart-type SR chart, because the highest ARL_0 that it can attain for subgroups of size 5 is 16 (see Bakir (2004), page 616). In addition, the *2-of-2* SR charts under the DR and KL schemes also can't attain the industry standard ARL_0 values; see Chakraborti and Eryilmaz (2007) Table 11, where it is shown that the highest ARL_0 value that the *2-of-2* DR scheme can attain for n = 5 is 271.15 when UCL = 15, whereas the *2-of-2* KL scheme can attain ARL_0 values of 136.00 and 526.34 for UCL = 13 and 15, respectively, for n = 5. Although the ARL_0 values of the Shewhart-type SR charts for UCL = 15 when n = 5 are far from the desired nominal *ARL* values, we include these charts for illustrative purposes. The values of the NPEWMA-SN and the NPEWMA-SR plotting statistics are presented in Table 5 along with the simulated observations. The control charts are shown in panels (a) – (d) of Figure 1.

Table 5. The observations, the NPEWMA-SN and NPEWMA-SR plotting statistics

Subgroup number (i)	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}	NPEWMA-SN	NPEWMA-SR
1	-0.442	1.236	1.486	-0.382	1.053	0.050	0.450
2	0.902	0.491	-1.383	2.488	0.558	0.198	0.778
3	2.023	-1.502	0.985	0.912	-1.314	0.238	0.789
4	1.264	3.340	1.372	-1.060	0.829	0.376	1.299
5	0.295	-0.227	0.586	0.413	1.435	0.507	1.884
6	0.212	0.082	-0.317	1.650	-0.077	0.532	2.040
7	0.752	0.118	0.521	-0.466	1.218	0.655	2.488
8	0.547	-0.453	-2.273	1.229	1.492	0.672	2.514
9	0.578	0.768	-0.372	0.777	0.227	0.789	2.938
10	0.174	0.440	-1.953	0.191	1.814	0.899	3.041
11	0.739	0.398	1.378	0.404	0.203	1.104	3.639
12	0.284	0.511	0.559	-0.237	1.465	1.199	4.107
13	-0.844	0.477	0.344	1.378	0.611	1.289	4.252
14	1.148	1.710	0.316	1.276	-0.156	1.375	4.689
15	1.212	1.652	0.643	-1.977	2.693	1.456	4.805

From panels (a) – (d) in Figure 1 we see that the plotting statistics of the nonparametric charts are the first to signal OOC at subgroup number 10, whereas the EWMA-$\overline{X}$ chart and the *1-of-1* SR chart signal later at subgroup number 11 and the *2-of-2* SR charts using the DR and KL signalling rules didn't signal at all. Although this is an example using simulated data, it shows that there are situations in practice where the nonparametric charts offer an effective alternative over available parametric charts.

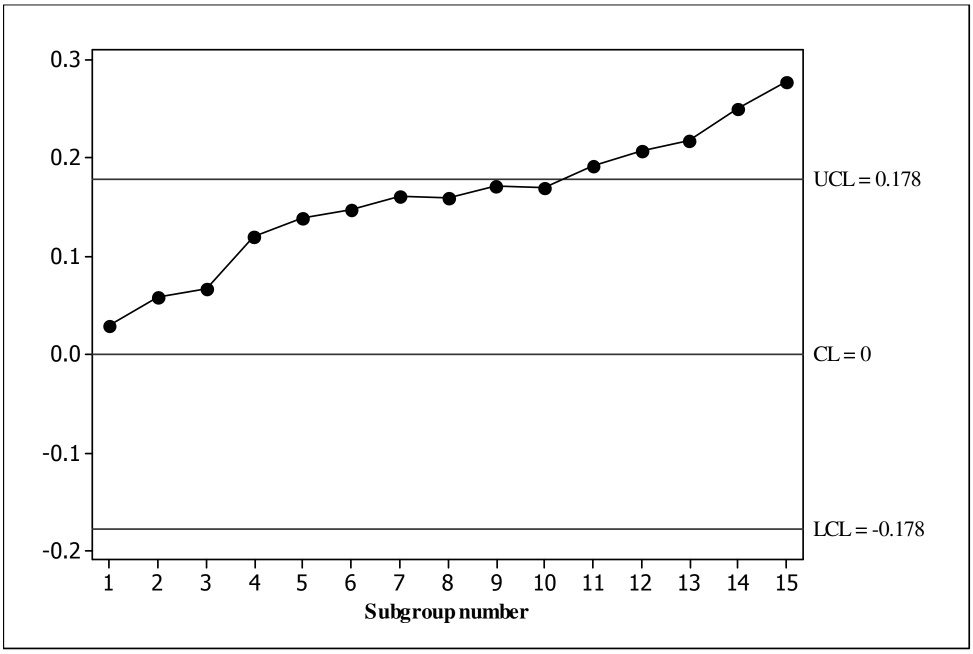

Figure 1a. The EWMA-$\overline{X}$ control chart for the Example with design parameters (λ,L) = (0.05, 2.488).

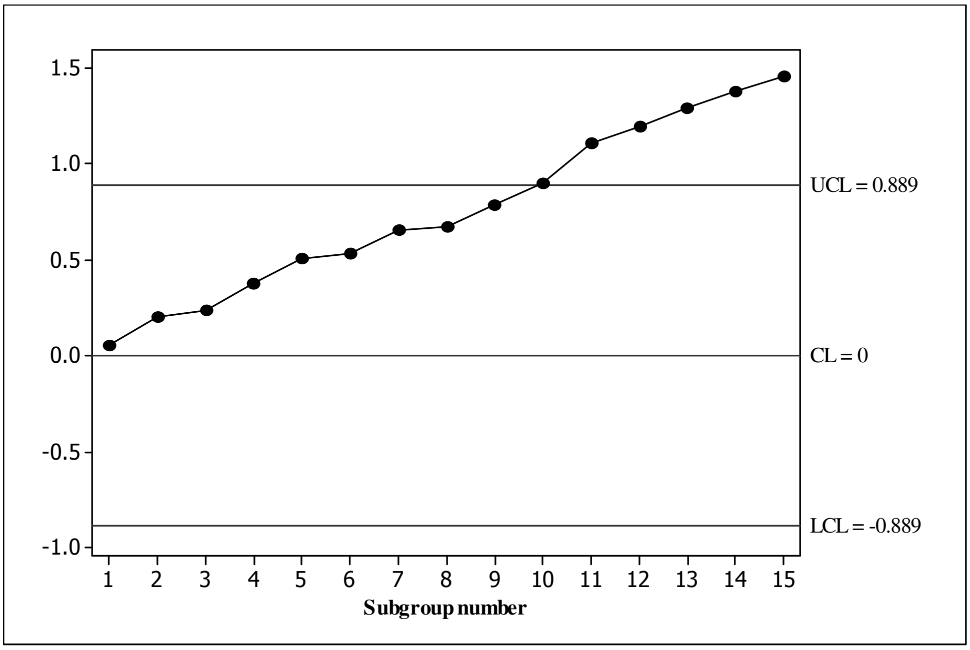

Figure 1b. The NPEWMA-SN control chart for the Example with design parameters (λ,L) = (0.05, 2.484).

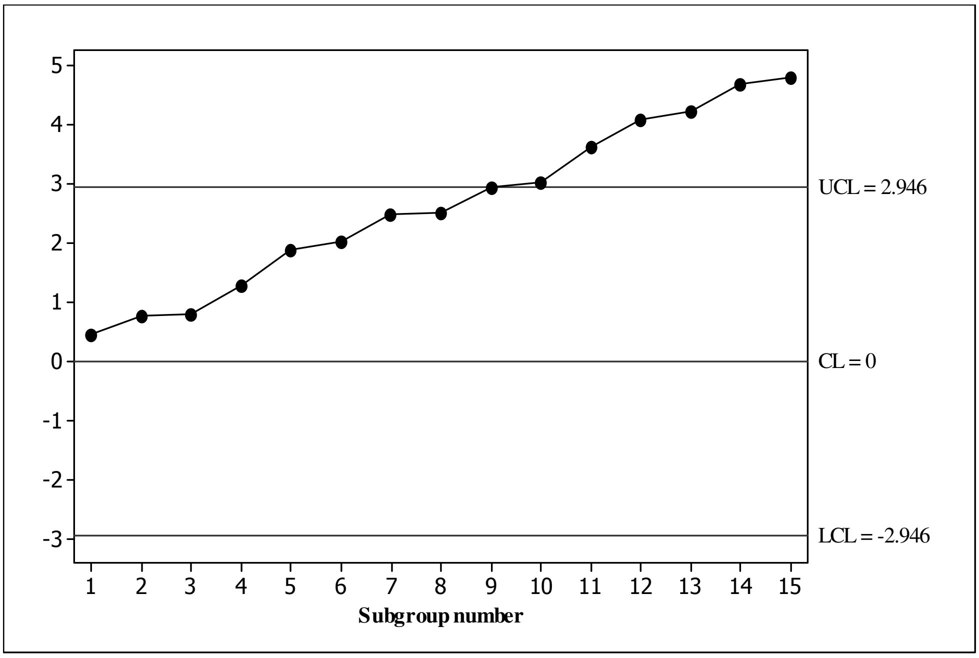

Figure 1c. The NPEWMA-SR control chart for the Example with design parameters $(\lambda, L) = (0.05, 2.481)$.

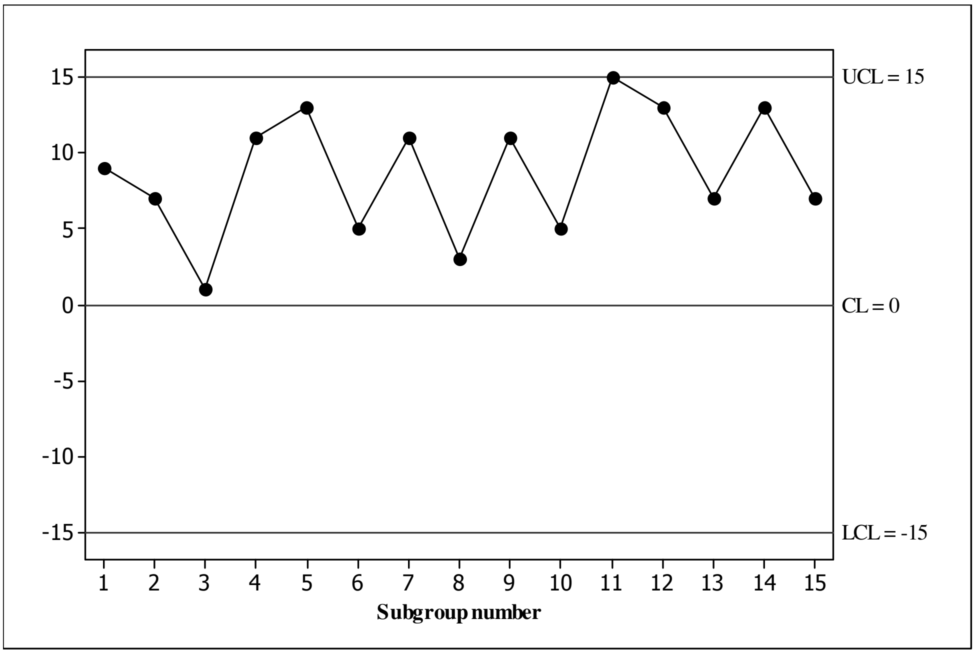

Figure 1d. The *1-of-1*, the *2-of-2* DR and the *2-of-2* KL Shewhart-type SR charts.

6. CONCLUDING REMARKS

EWMA charts take advantage of the sequentially (time ordered) accumulating nature of the data arising in a typical Statistical Process Control (SPC) environment and are known to be more efficient in detecting smaller shifts. The traditional parametric EWMA-$\overline{X}$ chart can lack in-control robustness and as such the corresponding false alarm rates can be a practical concern. Nonparametric EWMA charts offer an attractive alternative in such situations as they combine the inherent advantages of nonparametric charts (IC robustness) with the better small shift detection capability of EWMA-type charts. We study the nonparametric EWMA control chart based on the signed-rank statistic and its properties via the in-control and out-of-control run-length distribution using a Markov chain approach. It is seen that the nonparametric EWMA signed-rank chart performs as well as, and in some cases better than, the parametric EWMA chart, the nonparametric EWMA chart based on signs and the runs-rules enhanced Shewhart-type charts based on the signed-rank statistic. Hence these charts are recommended for use in practice.

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Table 3. The OOC performance characteristics of the run-length distribution for the EWMA-$\overline{\boldsymbol{X}}$, the NPEWMA-SN and the NPEWMA-SR charts for λ = 0.05, n = 10 and number of simulations = 100,000.

	EWMA-$\overline{X}$ chart with $\lambda = 0.05$ and L such that $ARL_0 \approx 500$					
	L	**Shift (number of standard deviations)**				
		0.5	**1**	**1.5**	**2**	**2.5**
***N*(0,1)**	***L*=2.613**	6.71 (1.89) 4, 5, 6, 8, 10	3.33 (0.64) 2, 3, 3, 4, 4	2.26 (0.44) 2, 2, 2, 3, 3	1.98 (0.15) 2, 2, 2, 2, 2	1.68 (0.47) 1, 1, 2, 2, 2
***t*(4)**	***L*=2.682**	30.94 (17.73) 11, 18, 27, 39, 65	11.76 (4.21) 6, 9, 11, 14, 20	7.29 (2.01) 5, 6, 7, 8, 11	5.34 (1.25) 4, 5, 5, 6, 8	4.26 (0.89) 3, 4, 4, 5, 6
***t*(8)**	***L*=2.640**	29.53 (16.99) 10, 18, 25, 37, 62	11.50 (4.22) 6, 9, 11, 14, 19	7.18 (2.05) 4, 6, 7, 8, 11	5.27 (1.27) 4, 4, 5, 6, 8	4.20 (0.90) 3, 4, 4, 5, 6
Laplace	***L*=2.666**	30.48 (17.58) 11, 18, 26, 38, 65	11.68 (4.27) 6, 9, 11, 14, 20	7.24 (2.05) 4, 6, 7, 8, 11	5.32 (1.27) 4, 4, 5, 6, 8	4.23 (0.89) 3, 4, 4, 5, 6
Logistic	***L*=2.635**	29.46 (17.00) 10, 17, 25, 37, 62	11.47 (4.22) 6, 8, 11, 14, 19	7.17 (2.05) 4, 6, 7, 8, 11	5.26 (1.27) 4, 4, 5, 6, 8	4.20 (0.90) 3, 4, 4, 5, 6
CN	***L*=2.656**	24.49 (18.26) 3, 11, 20, 33, 59	7.42 (4.73) 2, 4, 6, 10, 16	3.82 (2.20) 1, 2, 3, 5, 8	2.45 (1.28) 1, 2, 2, 3, 5	1.78 (0.85) 1, 1, 2, 2, 3
	NPEWMA-SN chart with $\lambda = 0.05$ and L such that $ARL_0 \approx 500$					
***N*(0,1)**	***L*=2.612**	9.01 (2.76) 5, 7, 9, 11, 14	4.78 (0.85) 4, 4, 5, 5, 6	3.65 (0.57) 3, 3, 4, 4, 4	3.15 (0.35) 3, 3, 3, 3, 4	3.01 (0.12) 3, 3, 3, 3, 3
***t*(4)**	***L*=2.612**	6.94 (1.76) 5, 6, 7, 8, 10	4.21 (0.69) 3, 4, 4, 5, 5	3.47 (0.53) 3, 3, 3, 4, 4	3.16 (0.37) 3, 3, 3, 3, 4	3.05 (0.22) 3, 3, 3, 3, 4
***t*(8)**	***L*=2.612**	8.08 (2.31) 5, 6, 8, 9, 12	4.53 (0.77) 3, 4, 4, 5, 6	3.58 (0.56) 3, 3, 4, 4, 4	3.17 (0.38) 3, 3, 3, 3, 4	3.04 (0.19) 3, 3, 3, 3, 3
Laplace	***L*=2.612**	6.56 (1.59) 5, 5, 6, 7, 9	4.29 (0.71) 3, 4, 4, 5, 5	3.57 (0.55) 3, 3, 4, 4, 4	3.22 (0.42) 3, 3, 3, 3, 4	3.07 (0.25) 3, 3, 3, 3, 4
Logistic	***L*=2.612**	8.00 (2.26) 5, 6, 8, 9, 12	4.53 (0.77) 3, 4, 4, 5, 6	3.59 (0.56) 3, 3, 4, 4, 4	3.18 (0.39) 3, 3, 3, 3, 4	3.04 (0.20) 3, 3, 3, 3, 3
CN	***L*=2.612**	8.61 (2.57) 5, 7, 8, 10, 13	4.65 (0.81) 4, 4, 5, 5, 6	3.59 (0.56) 3, 3, 4, 4, 4	3.14 (0.35) 3, 3, 3, 3, 4	3.02 (0.15) 3, 3, 3, 3, 3

	NPEWMA-SR chart with $\lambda = 0.05$ and L such that $ARL_0 \approx 500$					
		Shift (number of standard deviations)				
		0.5	**1**	**1.5**	**2**	**2.5**
***N*(0,1)**	***L*=2.610**	7.65 (1.97) 5, 6, 7, 9, 11	4.46 (0.58) 4, 4, 4, 5, 5	4.00 (0.07) 4, 4, 4, 4, 4	4.00 (0.00) 4, 4, 4, 4, 4	4.00 (0.00) 4, 4, 4, 4, 4
***t*(4)**	***L*=2.610**	6.51 (1.47) 5, 5, 6, 7, 9	4.27 (0.47) 4, 4, 4, 5, 5	4.01 (0.11) 4, 4, 4, 4, 4	4.00 (0.02) 4, 4, 4, 4, 4	4.00 (0.01) 4, 4, 4, 4, 4
***t*(8)**	***L*=2.610**	7.21 (1.77) 5, 6, 7, 8, 10	4.39 (0.55) 4, 4, 4, 5, 5	4.01 (0.09) 4, 4, 4, 4, 4	4.00 (0.01) 4, 4, 4, 4, 4	4.00 (0.00) 4, 4, 4, 4, 4
Laplace	***L*=2.610**	6.54 (1.51) 5, 5, 6, 7, 9	4.34 (0.52) 4, 4, 4, 5, 5	4.02 (0.13) 4, 4, 4, 4, 4	4.00 (0.02) 4, 4, 4, 4, 4	4.00 (0.00) 4, 4, 4, 4, 4
Logistic	***L*=2.610**	7.20 (1.77) 5, 6, 7, 8, 10	4.39 (0.55) 4, 4, 4, 5, 5	4.01 (0.10) 4, 4, 4, 4, 4	4.00 (0.01) 4, 4, 4, 4, 4	4.00 (0.00) 4, 4, 4, 4, 4
CN	***L*=2.610**	7.42 (1.87) 5, 6, 7, 8, 11	4.41 (0.56) 4, 4, 4, 5, 5	4.01 (0.08) 4, 4, 4, 4, 4	4.00 (0.01) 4, 4, 4, 4, 4	4.00 (0.00) 4, 4, 4, 4, 4