Right Endpoint of a distribution in Gumbel Domain of Attraction - Statistical Inference

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Extreme events are defined as extreme high (or low) values of whatever random characteristic we are interested in. These values play an important role because they may correspond to abnormal or dangerous operating conditions. Classical statistical inference techniques provide a good description of central behaviour, but not of extreme events.

Extreme Value Theory (EVT) gives a probabilistic framework to model extreme events. EVT describes the fluctuations of the maximum of a random sample with parent distribution function F,

$$M_n = \max(X1, \cdots, Xn).$$

Let x^F denote the (possibly infinite) right endpoint of F. That is,

$$x_F = \sup\{x : F(x) < 1\}.$$

Then one can show that M_n converges to x^F, with probability 1, as n approaches ∞.

In statistical analysis of rare events, the Generalized Extreme Value (GEV) is a unified version of the only three possible limits for the distribution of M_n, provided suitable normalization in scale and location, for a large enough sample size n. This is supported by EVT, which relies on the fundamental Theorem of Gnedenko(1943) on max-domains of attraction, comprising Fréchet, Weibull and Gumbel max-domains: Fréchet domain of attraction refers to dfs with polynomial decaying tails; Weibull domain to dfs light-tailed with finite right endpoint and Gumbel domain is the intermediate case which refers to a great variety of dfs possessing an exponential tail, having or not a finite right endpoint.

In fact, EVT is a general framework: the "heavy tail"case as been extensively addressed in the literature, but EVT can also deal with thin tail, or even "no tail"(finite x_F) cases. Less attention has yet been paid to the problem of assessing the presence of a distribution function F with finite x^F .

In the Gumbel max-domain setup, statistical inference for the (possible) finite right endpoint represents an important challenge for our knowledge into practical applications of real-world data sets in fields as in environmetrics, climatology, or sports.

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