Marshall-Olkin Family of Distributions and their applications in reliability theory, time series modeling and stress-strength analysis

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Abstract: In this paper we consider various Marshall-Olkin distributions and develop autoregressive minification processes with stationary marginals as exponential, Weibull, uniform, Pareto, Gumbel, Lomax etc. The applications in reliability modelling and stress-strength analysis are considered.Problems of estimation of parameters is addressed. Various properties are examined. Modelling with respect to a real data set is also done.Simulation studies are conducted to validate the theory.
Key words: Auto regressive modeling, Compounding, Geometric extreme stability, Reliability, Stress-Strength analysis

1. General Theory of Marshall-Olkin Distributions

Exponential, Weibull and Gamma are some of the important distributions widely used in reliability theory and survival analysis. These families and their usefulness are described by Cox and Oakes (1984). But these distributions have a limited range of behavior and cannot represent all situations found in applications. For example although the exponential distribution is often described as flexible, its hazard function is in fact restricted, being constant. The limitations of standard distributions often arouse the interest of researchers in finding new distributions by extending existing ones. The procedure of expanding a family of distributions for added flexibility or to construct covariate models is a well known technique in the literature. For instance the family of Weibull distributions contains exponential distribution and is constructed by taking powers of exponentially distributed random variables.Marshall and Olkin (1997) introduced a new method of adding a parameter into a family of distributions. According to them if $\bar{F}(x)$ denote the survival or reliability function of a continuous random variable X, then the usual device of adding a new parameter results in another survival function $\bar{G}(x)$ defined by

$$(1) \quad \bar{G}(x) = \frac{\alpha\bar{F}(x)}{1-\bar{\alpha}\bar{F}(x)} \quad ; -\infty < x < \infty; \alpha > 0, \bar{\alpha} = 1-\alpha$$

If $g(x)$ and $r(x)$ are the probability density function and hazard rate function corresponding to $\bar{G}$, then

$$(2) \quad g(x,\alpha) = \frac{\alpha f(x)}{[1-\bar{\alpha}\bar{F}(x)]^2} \quad ; -\infty < x < \infty, \alpha > 0, \bar{\alpha} = 1-\alpha \text{ and}$$

$$(3) \quad r(x,\alpha) = \frac{h(x)}{1-\bar{\alpha}\bar{F}(x)}$$

where $h(x)$ is the hazard rate corresponding to $f(x)$. From (3) it follows that $\frac{r(x,\alpha)}{h(x)}$ is increasing in x for $\alpha \geq 1$ and decreasing in x for $0 < \alpha \leq 1$. An extension to bivariate family of distributions is also introduced by Marshall and Olkin (1997). Let (X,Y) be a random vector with joint survival function $\bar{F}(x,y)$. Then

$$(4) \quad \bar{G}(x,y) = \frac{\alpha\bar{F}(x.y)}{1-\bar{\alpha}\bar{F}(x,y)} \quad ; x,y \geq 0,\ 0 < \alpha < 1,\ \bar{\alpha} = 1-\alpha$$

is a proper survival function. The family of distributions of the form (4) is called Marshall -Olkin bivariate family of distributions. Marshall-Olkin extended distributions offer a wide range of behavior than the basic distributions from which they are derived. The property that the extended form of distributions can have an interesting hazard function depending on the value of the added parameter α and therefore can be used to model real situation in a better manner than the basic distribution, resulted in the detailed study of Marshall-Olkin extended family of distributions by many researchers like Jose and Alice (2001,05), Alice and Jose(2002,03,04 (a,b,c)), Ghitany et al (2005), Jayakumar and Mathew (2006), Jayakumar and Kuttikrishnan (2006), Ghitany and Kotz (2007), Jose and Uma (2009) , Gupta et al (2010) and Jose et al (2010).

2. Reliability Applications

Sankaran and Jayakumar (2006) discussed the physical interpretation of Marshall-Olkin family of distributions using proportionate odds model. To analyse the life time data with covariates as odds ratio Bennet (1983) introduced the proportionate odds model as an alternative to the classical Cox's (1972) proportional hazard model in which the hazard rate of an individual with covariate **X** is given by

$$(5) \quad \lambda(t;x) = \lambda_0(t)exp(\mathbf{b}'X), t \geq 0$$

where **b** is the vector of unknown regression coefficients and $\lambda_0(t)$ is the base line hazard function. The covariates usually represents the heterogeneity in the population of life times in survival studies. Let X represents the lifetime of each individual in the population with a vector of p-covariates $\underline{z} = (z_1, z_2, \ldots, z_p)'$. The proportional odds model is defined by

$$(6) \quad \lambda_G(x/\underline{z}) = \lambda_F(x)[k(\underline{z})]^{-1}$$

where $\lambda_F(x)$ represents an arbitrary base line odds function with respect to the survival function$\bar{F}(x)$ defined by

$$(7) \quad \lambda_F(x) = \frac{F(x)}{\bar{F}(x)}$$

and $\lambda_G(x)$ is that corresponding to survival function $\bar{G}(x)$ and $k(\underline{z})$ is a non-negative function of $\underline{z}$ independent of time x. From (6) and (7) we get

$$(8) \quad \bar{G}(x/\underline{z}) = \frac{k(\underline{z})\bar{F}(x)}{1-(1-k(\underline{z}))\bar{F}(x)}$$

When $k(\underline{z}) = \alpha$, a constant then (8) becomes

$$\bar{G}(x) = \frac{\alpha\bar{F}(x)}{1-(1-\alpha)\bar{F}(x)}$$

which is the survival function of Marshall-Olkin family of distributions originating from $\bar{F}(x)$.
This result can be extended to bivariate case also. Let $\underline{X} = (X_1, X_2)$ be a random vector having an absolutely continuous distribution function $F(x_1, x_2)$ in the support of R_2^+. Let $\bar{F}(x_1, x_2) = P(X_1 > x_1, X_2 > x_2)$ be the survival function of $\underline{X}$. Sankaran and Jayakumar (2006) defined the odds function in the bivariate set up as

$$(9) \quad k_F(x_1, x_2) = \frac{1-\bar{F}(x_1, x_2)}{\bar{F}(x_1, x_2)}$$

where X_1 and X_2 represents life times of two components of a system, $k_F(x_1, x_2)$ is the ratio of the probability that the two components of a system will survive beyond (x_1, x_2) . The proportionate odds model is then given by

$$(10) \quad k_G(x_1, x_2) = [g(\underline{z})]^{-1} k_F(x_1, x_2)$$

where $\underline{z} = (z_1, z_2, \ldots, z_p)'$ is the vector of covariates associated with pair of units (X_1, X_2) and $g(\underline{z})$ is a non-negative function of $\underline{z}$, independent of x_1 and x_2. If $g(\underline{z}) = \alpha$, then from (9) and (10) we have,

$$(11) \quad \bar{G}(x_1, x_2) = \frac{\alpha\bar{F}(x_1, x_2)}{1-(1-\alpha)\bar{F}(x_1, x_2)}$$

which is the bivariate extension of Marshall-Olkin distribution.

3. Characterizations

3.1 Geometric extreme stability

For i.i.d random variables$X_1, X_2, \ldots, X_N$ with survival function(1.1) and suppose N has a geometric (p) distribution independent of $X_i's$. Then $U = min(X_1, X_2, \ldots, X_N)$ and $V = max(X_1, X_2, \ldots, X_N)$ have the distribution in the same family with $0 < \alpha = p \leq 1$ and $\alpha = \frac{1}{p} \geq 1$ respectively. Jose and Alice (2001) proved this property in the case of bivariate Marshall-Olkin family of distributions.

3.2 Compounding

Let $\bar{G}(x/\theta), -\infty < x < \infty, -\infty < \theta < \infty$, be the conditional survival function of a continuous random variable X given a continuous random variable Θ. Let Θ follows a distribution with probability density function $m(\theta)$. A distribution with survival function

$$\bar{G}(x) = \int_{-\infty}^{\infty} \bar{G}(x/\theta)m(\theta)d\theta \quad , -\infty < x < \infty$$

is called a compound distribution with mixing density $m(\theta)$. Compound distributions are helpful in obtaining new parameter families of distributions in terms of existing ones. Also when population items involve different risks they represent heterogeneous models. Ghitany (2005) proved this result for extended Pareto distribution. Ghitany et al (2007) derived the result for extended Weibull family, Ghitany et al (2005) derived the result for extended Lomax family and finally the result was proved for extended Linear exponential family by Ghitany and Kotz (2007).

3.3 Stress-strength Analysis

Gupta et al (2010) obtained various results on the MO family in the context of reliability modeling and survival analysis. The quantity $R = P(X < Y)$ where X denotes random stress and Y denotes random strength, is called the stress strength reliability in statistical literature. When stress exceeds strength the system fails. This measure of reliability is widely used in engineering problems under the banner reliability provided that random variables under consideration admit appropriate interpretation. Let X and Y are two independent random variables with survival function of the Marshall-Olkin family such that $\bar{G}_1(x, \alpha_1) = \frac{\alpha_1\bar{F}(x)}{1-(1-\alpha_1)\bar{F}(x)}$ and $\bar{G}_2(y, \alpha_2) = \frac{\alpha_2\bar{F}(y)}{1-(1-\alpha_2)\bar{F}(y)}$

$$(12) \quad P(X < Y) = P(Y > X) = \frac{\frac{\alpha_1}{\alpha_2}}{(\frac{\alpha_1}{\alpha_2} - 1)^2}[\frac{\alpha_1}{\alpha_2} - \ln(\frac{\alpha_1}{\alpha_2}) - 1].$$

Thus the reliability measure depends only on the tilt parameters α_1 and α_2. This can be used as a measure of the difference between two populations as well as the efficiency of one medicine over other.

4. Some Marshall-Olkin extended family of distributions and their properties

4.1 Marshall-Olkin Extended Exponential distribution-MOEE

Marshal and Olkin (1997) discussed the extended exponential distribution with 2 parameters with pdf

$$(13)\quad g(x,\alpha,\lambda) = \frac{\alpha\lambda e^{\lambda x}}{(e^{\lambda x}-\bar{\alpha})^2}\ ;\quad x>0, \alpha,\lambda>0, \bar{\alpha}=1-\alpha$$

$$(14)\quad HereMean = \frac{-\alpha\log\alpha}{\lambda\bar{\alpha}},\quad Median = \frac{\log(1+\alpha)}{\lambda}\ \text{ and } Mode=0\ \text{ if }\ \alpha\le 2 = \frac{1}{\lambda}\log(\alpha-1)\ \text{ if }\ \alpha\ge 2$$

The hazard rate is

$$r(x,\alpha,\lambda) = \frac{\lambda e^{\lambda x}}{e^{\lambda x}-\bar{\alpha}} \text{ and } \lim_{x\to\infty} r(x,\lambda,\alpha) = \lambda$$

is not a constant but is bounded and continuous in the parameters like the gamma distribution.

4.2 Marshall-Olkin Extended Weibull distribution - MOEW

Marshall and Olkin (1997) introduced the extended form of three parameter Weibull distribution with p.d.f,

$$(15)\quad g(x,\alpha,\lambda,\beta) = \frac{\alpha\beta\lambda^\beta x^{\beta-1}e^{(\lambda x)^\beta}}{[e^{(\lambda x)^\beta}-\bar{\alpha}]^2}, x>0, \alpha,\beta,\lambda>0$$

Here $E(X^s) = \frac{s\alpha}{\beta}\sum_{j=0}^{\infty}\frac{\bar{\alpha}^j}{(j+1)^{s/\beta}}\Gamma\frac{s}{\beta}$ $\quad(|\bar{\alpha}|\le 1)$ and the hazard rate is

$$r(x,\alpha,\lambda,\beta) = \frac{\lambda\beta(\lambda x)^{\beta-1}e^{(\lambda x)^\beta}}{e^{(\lambda x)^\beta}-\bar{\alpha}}$$

Alice and Jose (2005) introduced Marshall-Olkin Semi Weibull distribution.

Compounding: With $\bar{G}(x/\theta) = e^{\theta[1-e^{(\lambda x)^\beta}]}$ and mixing density $m(\theta) = \alpha e^{-\alpha\theta}$, Ghitany et al (2005) expressed MOEW as a compound distribution.

Naik, Jose and Ristic (2008) introduced Marshall-Olkin q-Weibull distribution(MO-q-W). The pdf of MO-q-W is obtained as

$$(16)\quad g_1(x,p,q,\alpha,\lambda) = \frac{p\alpha\lambda^\alpha(2-q)x^{\alpha-1}(1+(q-1)(\lambda x)^\alpha)^{\frac{-1}{q-1}}}{(1-(1-p)(1+(q-1)(\lambda x)^\alpha)^{\frac{q-2}{q-1}})^2}\quad x>0, 1<q<2 \text{ and}$$

$$(17)\quad g_2(x,p,q,\alpha,\lambda) = \frac{p\alpha\lambda^\alpha(2-q)x^{\alpha-1}(1-(1-q)(\lambda x)^\alpha)^{\frac{1}{1-q}}}{(1-(1-p)(1-(1-q)(\lambda x)^\alpha)^{\frac{2-q}{1-q}})^2},\quad 0<x<(\lambda(1-q)^{1/\alpha})^-1,\quad for\ q<1$$

The hazard rate function is

$$(18)\quad r_1(x,p,q,\alpha,\lambda) = \frac{\alpha\lambda^\alpha(2-q)x^{\alpha-1}[1+(q-1)(\lambda x)^\alpha]^{-1}}{1-(1-p)[1+(q-1)(\lambda x)^\alpha]^{\frac{q-2}{q-1}}}, 1¡q¡2$$

Jose, Ristic and Ancy (2009) introduced type 2 Marshall-Olkin Bivariate Weibull as an extension to the bivariate exponential distribution of Marshall-Olkin (1967) .

4.3 Marshall-Olkin extended Pareto distribution - MOEP

Alice and Jose (2003) introduced Marshall-Olkin extended semi Pareto model (MOSP) for Pareto type III with survival function

$$\bar{G}(x,\alpha) = \frac{1}{1+\frac{1}{\alpha}\psi(x)}\ ;\ x\ge 0, \alpha>0$$

and established its geometric extreme stability. Ghitany (2005) introduced Marshall-Olkin extension of Pareto type I distribution.

Compounding We can express MOEP with $\bar{G}(x) = \frac{\alpha}{(\frac{x}{\beta})^\gamma-\bar{\alpha}}$ as a compounding distribution with $\bar{G}(x/\theta) = e^{\{-[(\frac{x}{\beta})^\gamma-1]\theta\}}$ and mixing density$m(\theta) = \alpha e^{-\alpha\ \theta}$

4.4 Marshall -Olkin Extended Logistic Distribution- MOEL

Alice and Jose(2005) generalised standard logistic distribution by Marshall-Olkin technique yielding the pdf

$$(19)\quad g(x,\alpha,\lambda,\theta) = \frac{\alpha\lambda\theta e^{\lambda x}}{(\alpha+\theta e^{\lambda x})^2}\ ;-\infty<x<\infty, \alpha,\theta,\lambda>0$$

Here $Mean = \log\left(\frac{\alpha}{\theta}\right)^{1/\lambda}$ and the hazard rate is given by

$$(20)\quad r(x,\alpha,\lambda,\theta) = \frac{\lambda\theta e^{\lambda x}}{\alpha+\theta e^{\lambda x}}, -\infty < x < \infty, \alpha,\theta,\lambda > 0$$

The Marshall-Olkin semi-logistic distribution is also introduced by them as generalizations.

4.5 Marshall-Olkin Extreme value distributions

The pdf and hazard rate function for Marshall-Olkin Gumbel(maximum) distribution are derived respectively as

$$(21)\quad g(x,\alpha,\lambda,\delta) = \frac{\alpha e^{-(\frac{x-\lambda}{\delta})} e^{-e^{-}(\frac{x-\lambda}{\delta})}}{\delta[1-(1-\alpha)(1-e^{-e^{-(\frac{x-\lambda}{\delta})}})]^2}, -\infty < x < \infty, \alpha,\lambda,\delta > 0 \text{ and}$$

$$(22)\quad r(x,\alpha,\lambda,\delta) = \frac{e^{-(\frac{x-\lambda}{\delta})} e^{-e^{-}(\frac{x-\lambda}{\delta})}}{\delta[1-(1-\alpha)(1-e^{-e^{-(\frac{x-\lambda}{\delta})}})][1-e^{-e^{-}(\frac{x-\lambda}{\delta})}]}$$

The p.d.f and hazard rate function for Marshall-Olkin Gumbel(minimum) distribution are respectively

$$(23)\quad g(x,\alpha,\lambda,\delta) = \frac{\alpha e^{-(\frac{\lambda-x}{\delta})} e^{-e^{-}(\frac{\lambda-x}{\delta})}}{\delta[1-(1-\alpha)(1-e^{-e^{-(\frac{\lambda-x}{\delta})}})]^2}, -\infty < x < \infty, \alpha,\lambda,\delta > 0 \text{ and}$$

$$(24)\quad r(x,\alpha,\lambda,\delta) = \frac{e^{-(\frac{\lambda-x}{\delta})} e^{-e^{-}(\frac{\lambda-x}{\delta})}}{\delta[1-(1-\alpha)(1-e^{-e^{-(\frac{\lambda-x}{\delta})}})][1-e^{-e^{-}(\frac{\lambda-x}{\delta})}]}$$

The p.d.f and hazard rates of Marshall-Olkin *Fréchet* Maximum (MO-FRMX) and Marshall-Olkin *Fréchet* minimum (MO-FRMN) are derived by Alice and Jose (2005).

4.6 Marshall-Olkin Burr Distribution

Exponentiating the survival function of Marshall-Olkin semi Pareto type III distribution, a more generalised family of distribution can be obtained. The resulting expression will be

$$(25)\quad \bar{G}(x,\alpha,\beta,\gamma) = \left(\frac{1}{1+\frac{1}{\alpha}\psi(x)}\right)^{\gamma} \alpha,\beta,\gamma > 0$$

Jayakumar and Mathew (2006) referred to this as Marshall-Olkin semi Burr distribution. As a special case to (25) they defined Marshall-Olkin Burr distribution MOB(α,β,γ). Ghitany et al (2007) studied the properties of the extended form of Lomax distribution with one more parameter added. Ghitany and Kotz (2007) investigated the reliability properties of extended Marshall-Olkin failure rate distribution(linear exponential distribution).

5. Application in Auto-regressive Time series Modeling

Time series modeling is finding its applications in diversified fields today. There are two approaches to time series analysis namely. the time domain approach and frequency domain approach. The time domain approach focuses on modeling some future values of a time series as a parametric function of the current and the past values. The frequency approach assumes primary interest in time series analysis related to periodic relations found naturally in most data. One of the simplest and widely used time series models is the autoregressive models and autoregressive process of appropriate orders are extensively used for modeling time series data. Lewis and Mckenzie (1991) introduced and discussed various minification processes having structure

$$X_n = k\ min(X_{n-1}, \epsilon_n)$$

Jose and Alice (2005), Naik and Jose (2008), Jose et al (2009) had studied various minification processes with respect to Marshall-Olkin extended distributions in detail.

5.1 MIN AR(1) model-I

Consider an AR(1) structure given by

$$(26)\quad X_n = \begin{cases} \varepsilon_n & \text{with probability } p \\ \min(X_{n-1},\varepsilon_n) & \text{with probability } 1-p \end{cases}$$

where $\{\varepsilon_n\}$ is a sequence of independent and identically distributed random variables independent of $\{X_n\}$. Then $\{X_n\}$ is stationary Markovian AR(1) process with MO distribution as marginal. The converse is also true. Jose and Alice (2005) proved this result for the Marshall-Olkin extended distributions like MOEE, MOEW, MOEP and MO-FRMX and utilising the result the sample path is explored.

5.2 MIN AR(1) model-II

A more general structure which allows probabilistic selection of process values, innovations and combinations of both is given below.
Consider an AR(1) structure given by

$$(27)\quad X_n = \begin{cases} X_{n-1} & \text{with probability } p_2 \\ \varepsilon_n & \text{with probability } \quad p_1(1-p_2) \\ \min(X_{n-1}, \varepsilon_n) & \text{with probability } \quad (1-p_1)(1-p_2) \end{cases}$$

where $\{\varepsilon_n\}$ is a sequence of independent and identically distributed random variables independent of $\{X_n\}$. Then the process is stationary Markovian with MO distribution as marginal. Jose and Alice (2005) studied the structures (26) and (27) with respect to the MO familes like MOEE, MOEW, MOEP,MO-FRMX etc.

5.3 MAX-MIN AR(1) model-I

Consider the AR(1)structure given by

$$(28)\quad X_n = \begin{cases} \max(X_{n-1}, \varepsilon_n) & \text{with probability } \quad p_1 \\ \min(X_{n-1}, \varepsilon_n) & \text{with probability } \quad p_2 \\ X_{n-1} & \text{with probability } \quad 1-p_1-p_2 \end{cases}$$

subject to the conditions $0 < p_1, p_2 < 1, p_2 < p_1$ and $p_1 + p_2 < 1$ where $\{\varepsilon_n\}$ is a sequence of i.i.d random variables independently distributed of X_n. Then$\{X_n\}$ is stationary Markovian AR(1) max-min process with stationary marginal distribution$\bar{F}_X(x)$ if and only if $\{\varepsilon_n\}$follows MO distribution and vice versa.

5.4 MAX-MIN AR(1) model-II

Finally we consider a more general Max-Min process which includes maximum, minimum as well as the innovations and the process having the AR(1) structure given by

$$(29)\quad X_n = \begin{cases} \max(X_{n-1}, \varepsilon_n) & \text{with probability} p_1 \\ \min(X_{n-1}, \varepsilon_n) & \text{with probability} p_2 \\ \varepsilon_n & \text{with probability } p_3 \\ X_{n-1} & \text{with probability} 1-p_1-p_2-p_3 \end{cases}$$

subject to the conditions $0 < p_1, p_2, p_3 < 1$ and $p_1 + p_2 + p_3 < 1$ where $\{\varepsilon_n\}$ is a sequence of i.i.d random variables independently distributed of X_n. Then $\{X_n\}$ is stationary Markovian AR(1)max-min process with stationary marginal distribution $\bar{F}_x(x)$ if and only if $\{\varepsilon_n\}$ follows MO distribution and vice versa. Naik et al(2008) discussed the structures (28) and (29) with respect to Marshall-Olkin q-Weibull distribution.

6. Extended distributions from characteristic function

Jayakumar et al (2006) introduced a method of expanding a family of distributions with a proper characteristic function by adding a parameter to it by the Marshall-Olkin technique to get the characteristic function of the new family. If $\phi(t)$ is the characteristic function of the random variable X, then

$$(30)\quad \psi(t) = \frac{\alpha\phi(t)}{1-(1-\alpha)\phi(t)}; \quad \alpha > 0, \bar{\alpha} = 1-\alpha$$

is a proper characteristic function and the corresponding distribution can be regarded as a Marshall-Olkin extended form.

6.1 Marshall-Olkin Asymmetric Laplace distributions- (MOAL)

Jayakumar and Kuttykrishnan (2006) defined the characteristic function of MOAL as

$$(31)\quad \psi(t) = [1 + \frac{1}{\alpha}(\sigma^2 t - i\mu t)]^{-1}, \alpha > 0$$

They also developed MOAL processes with the structure

$$(32)\quad X_n = \begin{cases} \varepsilon_n & \text{with probability } \frac{1}{\beta} \\ \min(X_{n-1}, \varepsilon_n) & \text{with probability } 1-\frac{1}{\beta} \end{cases}$$

where $\beta > 1$ and $\{\epsilon_n\}$ is i.i.d random variables independent of X_n.

6.2 Generalised Linnik Distribution

Jose and Uma (2009) gave an extension to the characteristic function of Linnik distribution by Marshall-Olkin scheme called generalised Linnik distribution (GL). The characteristic function of GL(ν, α, β) is

$$(33)\quad \psi(t) = \frac{\beta}{(1+|t|^\alpha)^\nu + \beta - 1}, \nu > 0, 0 < \alpha \leq 2, \beta > 0$$

The extended distribution is proved to be self decomposable and is closed under geometric compounding. They also developed two autoregressive models with GL(ν, α, β) as marginal distributions.

6.3 Marshall-Olkin Mittag-Leffler distribution:MOML

Jose and Uma (2009) applied Marshall-Olkin method of extension to the characteristic function of Mittag-Leffler family of distributions to get the characteristic function of the new family referred to as Marshall-Olkin Mittag -Leffler distribution $MOML(\alpha, \beta)$ given by $\psi(t) = \frac{\beta}{\beta+s^{\alpha}} \quad ; \beta > 0$

6.4 Conclusion

From the discussions given above, it is clear that Marshall -Olkin family of distributions provide a flexible class of distributions having applications in various areas like distribution theory, reliability theory, stress strength analysis, time series modeling etc. More over these distributions possess properties like geometric extreme stability including mixing properties which make them appropriate compound distributions. The extension to the bivariate and multivariate cases are easy and direct.

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RÉSUMÉ (ABSTRACT)

Dr. Kanichukattu Korakutty Jose (K.K. Jose) is presently Principal of St.Thomas College Palai, Kerala,India. He was formerly Professor and Head of the Department of Statistics here and is an elected member of ISI. He has published more than 100 research papers in national/international journals, authored 5 research level books and successfully supervised 12 research scholars leading to Ph.D. His areas of research include Distribution Theory, Time Series Modeling,Reliability Analysis, Process Control, Acceptance Sampling, etc.