

Structural changes in models for I(1) and I(2) converging series: the case of PPP

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Introduction

The procedure suggested by Lee and Strazicich (2003) and applied in Strazicich et al. (2004) for assessing convergence between incomes of different OECD countries, aims at testing for a unit root, but allowing for possible breaks in the levels and in the trends of the series. In contrast to other testing procedures that reject the null of a unit root in the presence of breaks, they show that their testing methodology is not subject to this kind of spurious rejection and therefore, if the null is rejected the series is indeed stationary. They apply their methodology in order to test for stochastic convergence in incomes in OECD countries over a very long period of observation, during which structural breaks have effectively occurred (Strazicich, Lee and Day, 2004). Rejection of the null of a unit root in relative incomes, even with significant structural breaks in the observed series, means that shocks to relative incomes are temporary and implies that incomes are converging according to the definition of Bernard and Durlauf (1995).

The aim of the present paper is to empirically investigate the purchasing power parity (PPP) condition between three Eastern European economies and the euro-area economy as a convergence process. We consider, as the period of observation, the recent years corresponding to the catching-up process of these economies towards their accession to the EMU. The idea is that the persistent deviations from PPP that we observe may be due to the fact that an I(2) common stochastic trend is driving the variables towards convergence to the PPP condition, and not just an I(1) common trend as assumed by Bernard and Durlauf (1995).

Absolute PPP is usually written as:

$$ppp_t^* = (p_t^* - p_t^{eu}) - s_t^*$$

where p_t^* is the log of the domestic price level, p_t^{eu} is the log of the euro-area price level and s_t^* is the log of the domestic nominal exchange rate with the euro as the base currency. The absolute version of the PPP requires that ppp_t^* is stationary, or a mean-reverting process, where ppp_t^* can be interpreted as the real exchange rate.

We apply the testing procedure of Lee and Strazicich to each of the three country-pairs, Czech Republic/euro-area, Hungary/euro-area and Poland/euro-area¹, in order to establish whether the ppp_t^* series do reject the null hypothesis of a unit root in the presence of possible structural breaks, thus indicating a probable convergence between the price differentials and the exchange rate over the observation period. The observation period is not very long and characterized by the recent economic crisis that could have caused structural breaks in the observed series.

Whether there are significant structural breaks or not, in any case the results of the application of the testing procedure of Lee and Strazicich show that the ppp_t^* series do not reject the null hypothesis and

¹ The dataset for each country - Czech Republic, Hungary and Poland - is made by monthly time series from January 1999 to January 2011: the domestic Harmonised Index of Consumer Price (HICP), the euro-area HICP, the nominal exchange rate expressed as the home currency price relative to the euro. The source of the data is Eurostat.

therefore can be considered as non stationary and not converging for each country pair, according to this approach, which essentially aims to establish the mean-reversion of the series of interest, assuming that the series are generated by an I(1) process. But the dynamics of adjustment towards the PPP could be much slower than would be expected by the model implied by this procedure. In particular, support for PPP is very dependent on the sample period chosen and it could take a longer time to get statistically significant mean reversion, especially when there could have been structural breaks disturbing the convergence process. As Bernard and Durlauf (1995, p. 100) observe: " ... If the countries in our sample start at different initial conditions and are converging to, but are not yet at a steady-state ... then the available data may be generated by a transitional law of motion, rather than by an invariant stochastic process ... unit root tests may erroneously accept a no-convergence null ... "

A slow adjustment of relative prices and exchange rates can be considered as an indication that deviations from equilibrium exhibit a pronounced persistence typical of system driven by common stochastic trends of the I(2) type. Therefore, in order to establish stochastic relative convergence of time series, we should take into account also the possibility that the series are generated by I(2), or near I(2), processes.

The statistical model

The reference statistical I(2) model, within the class of Cointegrated Vector Auto-Regressive (CVAR) models, would then be the following for the vector \mathbf{x}_t of p observed variables, in the case of a VAR(3) (Juselius, 2006, pp. 312-313):

$$\Delta^2 \mathbf{x}_t = -\Gamma_2 \Delta^2 \mathbf{x}_{t-1} + \Gamma \Delta \mathbf{x}_{t-1} + \Pi \mathbf{x}_{t-1} + \Phi \mathbf{D}_t + \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N_p(\mathbf{0}, \Omega) \quad (1)$$

The existence of cointegration between the variables implies that both matrices Π and Γ are of reduced rank and can be written as:

$$\Pi = \boldsymbol{\alpha} \boldsymbol{\beta}', \quad \text{where } \boldsymbol{\alpha}, \boldsymbol{\beta} \text{ are } p \times r$$

$$\boldsymbol{\alpha}'_{\perp} \Gamma \boldsymbol{\beta}_{\perp} = \boldsymbol{\xi} \boldsymbol{\eta}', \quad \text{where } \boldsymbol{\xi}, \boldsymbol{\eta} \text{ are } (p-r) \times s_1$$

where $r < p$ is the cointegration rank of the system and $\boldsymbol{\alpha}_{\perp}$ and $\boldsymbol{\beta}_{\perp}$ are $p \times (p-r)$ orthogonal to $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. The first reduced rank condition is associated with the variables in levels and the second with the variables in differences, moreover, the first decomposes the vector process into r cointegrating relations and $(p-r)$ common stochastic trends and the second decomposes the latter into s_1 first order stochastic trends and $s_2 = p-r-s_1$ second order stochastic trends. As shown in Johansen (1997), the I(2) model in (1) contains $(p-s_2)$ cointegration relations of the following form:

$$\boldsymbol{\beta}' \mathbf{x}_t \sim I(1), \quad r \text{ relations, which can become stationary by polynomial cointegration, } \boldsymbol{\beta}' \mathbf{x}_t + \boldsymbol{\delta}' \Delta \mathbf{x}_t \sim I(0),$$

$$\boldsymbol{\beta}'_{\perp} \mathbf{x}_t \sim I(1), \quad s_1 \text{ relations, which can become stationary by differencing, } \boldsymbol{\beta}'_{\perp} \Delta \mathbf{x}_t \sim I(0).$$

The vector of observed variables $\mathbf{x}_t' = (p_t^*, p_t^{eu}, s_t^*)$ has been augmented in our empirical analysis in order to allow for deterministic trend components in the cointegrating relations and also for possible linear broken trends modelling the structural changes that could occur during the catching-up process of the economies, especially during the recent economic crisis. These trend components need to be restricted in such a way that they do not cumulate to quadratic or cubic trends.

In a compact form, the cointegrated I(2) model we estimate is the following (Johansen et al., 2010, p.120):

$$\Delta^2 \mathbf{x}_t = \boldsymbol{\alpha} (\tilde{\boldsymbol{\beta}}' \tilde{\mathbf{x}}_{t-1} + \tilde{\boldsymbol{\delta}}' \Delta \tilde{\mathbf{x}}_{t-1}) + \boldsymbol{\omega}' \tilde{\boldsymbol{\tau}}' \Delta \tilde{\mathbf{x}}_{t-1} + \Phi \mathbf{D}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N_p(\mathbf{0}, \Omega), \quad (2)$$

where $\tilde{\mathbf{x}}_t' = (p_t^*, p_t^{eu}, s_t^*, t, t_1, t_2)$ and the number r of stationary polynomially cointegrating relations - the relations within round brackets in (2) - the number s_1 of I(1) common stochastic trends and the number s_2 of I(2) ones among the $(p-r)$ non stationary components, are determined through appropriate test procedures.

The empirical analysis

For each pair of countries² we estimated by maximum likelihood an unrestricted VAR model³ and then we tested the cointegration rank, first in the I(1) case, computing the LR Joahansen's trace statistic, and then in the I(2) case, through a LR procedure (Nielsen, Rahbek, 2007), where the trace test is calculated for all possible values of r and s_1 .

Czech Republic/euro-area

The structural break and the corresponding broken trend has been determined by a sequential search performed using a LR test based on the null hypothesis of long-run exclusion of the broken trend from the system of variables. The reason is that Lee and Strazicich (2003) procedure is not very helpful within CVAR models, because it doesn't take into account long-run dynamics as well as short-run dynamics. The broken linear trend is specified in 2008:6, which corresponds to a change in the relative prices and in the exchange rate. After having estimated the unrestricted model⁴, we computed the LR test for cointegration rank in the I(1) model, but the identified significant cointegration relation showed a clear non-stationary behaviour. Secondly we computed the test of the joint hypothesis (r, s_1, s_2) for all values of r, s_1 and s_2 and the first non rejection was for $\{r=2, s_1=0, s_2=1\}$, with a p -value of 0.368. These implies two unit roots in the system corresponding to a single I(2) stochastic trend. Having set the cointegration indices r and s_2 , quite surprisingly, the iterative procedure converged to the final estimates in few iterations. Then we tested the hypotheses that the broken linear trend as well as the trend itself are long-run excludable from $\tilde{\tau}$ (Johansen et al., 2010, p.124) both hypotheses were very largely rejected.

Trying to identify the long-run structure underlying the two relations, the first interesting hypothesis is whether we can find a stationary polynomial cointegrating relation involving the 'strong form' of PPP. The final identified structure is given by the following normalized relations, accepted with a p -value of 0.161:

$$\begin{aligned}\hat{\beta}'_1 \tilde{x}_t + \hat{\delta}'_1 \Delta \tilde{x}_t &= p_t^{Czk} - p_t^{eu} - s_t^{Czk} + 80.949 \Delta p_t^{Czk} + 13.884 \Delta p_t^{eu} + 67.054 \Delta s_t^{Czk} - 0.003t + \dots \\ \hat{\beta}'_2 \tilde{x}_t + \hat{\delta}'_2 \Delta \tilde{x}_t &= -0.450 p_t^{Czk} + p_t^{eu} + 0.337 s_t^{Czk} - 27.159 \Delta p_t^{Czk} - 4.662 \Delta p_t^{eu} - 22.487 \Delta s_t^{Czk} + 0.0001t_{08:6} + \dots\end{aligned}$$

In the first relation we find evidence of 'strong form' of PPP, which is made stationary around the trend component by the inflation differential between the two countries and by the changes in the exchange rate. The significant negative trend component could due to the different deterministic trends characterizing the relative priced and the nominal exchange rate, though driven by the same stochastic trend. The existence of this trend could be interpreted as a possible convergence in act between the two variables.

The second relation doesn't have a direct economic interpretation, it's a relation between the level of prices in both countries and the nominal exchange rate.

² A similar analysis has been performer by Bacchiocchi and Fanelli (2005), where they consider as country-pairs France/USA, Germany/USA and UK/USA. The difference with our analysis is that they do not deal with the problem of convergence and of structural breaks.

³ The empirical analysis was performed using the subroutine CATS, which needs the software RATS to be run (Dennis, 2006).

⁴ The number $K=2$ of lags chosen is suggested by misspecification testing of the baseline VAR. As a consequence, in the model (1) there will be no $\Delta \mathbf{x}_{t-1}^2$ term. \mathbf{D}_t is a vector containing three impulse dummies, which take the value one in 2008:1, 2009:2 and 2010:1, respectively. Shift dummies are introduced by the program when specifying broken trends in the cointegrating relations. The misspecification tests for the unrestricted VAR(2) model with dummies take the following values: the $LM(1)$ test for first order autocorrelation is equal to 9.635 with a p -value of 0.381, while the $LM(2)$ test for second order autocorrelation is equal to 21.577 with a p -value of 0.010; the LM tests show no ARCH effects, while the normality test shows some problems in the Czech prices.

The estimated adjustment coefficients - not reported for reasons of space coefficients - show a slow and borderline significant adjustment of Czech prices to both long-run equilibrium relations.

The I(2) common stochastic trend, which can be considered as the force driving this system of variables, results to be made up mainly by twice cumulated shocks to Czech prices and to the exchange rate.

Hungary/euro-area

As for the Czech Republic, the structural break and the corresponding broken trend has been determined by the sequential test procedure based on the null hypothesis of long-run exclusion of the broken trend from the system of variables. The broken linear trend is specified in 2007:8, which corresponds to a change in the trend of the exchange rate. After having estimated the unrestricted model⁵ and computed the LR test for cointegration rank in the I(1) model with no relevant interesting cointegrating relation, we computed the test of the joint hypothesis (r, s_1, s_2) and the first non rejection was for $\{r=2, s_1=0, s_2=1\}$, with a p-value of 0.261. Then we tested the hypotheses that the broken linear trend and the trend are long-run excludable they were very largely rejected.

Trying to identify whether the polynomial cointegrated relations give evidence of the 'strong form' of PPP, we got the final identified structure given by the following normalized relations, accepted with a p-value of 0.713:

$$\begin{aligned}\hat{\beta}'_1 \tilde{x}_t + \hat{\delta}'_1 \Delta \tilde{x}_t &= p_t^{Hum} - p_t^{eu} - s_t^{Hum} + 13.482 \Delta p_t^{Hum} + 0.004 \Delta p_t^{eu} + 13.466 \Delta s_t^{Hum} + 0.004 t_{07:8} - 0.002 t + \dots \\ \hat{\beta}'_2 \tilde{x}_t + \hat{\delta}'_2 \Delta \tilde{x}_t &= -0.460 p_t^{Hum} + p_t^{eu} + 0.461 s_t^{Hum} - 6.162 \Delta p_t^{Hum} - 0.006 \Delta p_t^{eu} - 6.145 \Delta s_t^{Hum} - 0.001 t_{07:8} + \dots\end{aligned}$$

Again in the first relation we find significant evidence of 'strong form' of PPP, which is made stationary by the inflation differential between the two countries and by the changes in the exchange rate. The trend components show coefficients with opposite sign after 2007:8, as if the deterministic trend changed direction at the time of the structural break. The resulting deterministic component is just a constant implying a relative convergence between the difference in prices and the nominal exchange rate, when the differential of inflations and the movements in the exchange rate are taken into account.

As for the Czech Republic, the second relation is a relation between the prices and the nominal exchange rate.

The estimated adjustment coefficients show that prices in Hungary adjust significantly to both long-run equilibrium relations.

The I(2) common stochastic trend results to be made up mainly by twice cumulated shocks to euro-area prices and to the exchange rate.

Poland/euro-area

In this dataset the series of relative prices show changes in the slope of its trend corresponding to 2001:1 and 2008:2, while the exchange rate show big persistent swings but no significant trend. Therefore we specified two broken linear trends modelling the two changes. After having estimated the unrestricted model⁶ and computed the LR test for cointegration rank in the I(1) model with no relevant interesting

5 \mathbf{D}_t is a vector containing two impulse dummies, which take the value one in 2004:1, the date of the accession to the European union, and in 2010:1, respectively. Shift dummies are added by the program. The misspecification tests for the unrestricted VAR(3) model with dummies take the following values: the $LM(1)$ test for first order autocorrelation is equal to 14.977 with a p-value of 0.092, while the $LM(2)$ test for second order autocorrelation is equal to 14.955 with a p-value of 0.092; the LM tests show no ARCH effects, while the normality test shows some marginal problems.

6 Given the number $K=2$ of lags, in the model (1) there will be no $\Delta \mathbf{x}_{t-1}^2$ term. \mathbf{D}_t is a vector containing just one impulse dummy, which take the value one in 2010:1. Shift dummies are added by the program. The

cointegrating relation, we computed the test of the joint hypothesis (r, s_1, s_2) and the first non rejection was for $\{r=2, s_1=0, s_2=1\}$, with a p-value of 0.804. Having set the cointegration indices r and s_2 , quite surprisingly, the iterative procedure converged to the final estimates in few iterations. Then we tested the hypotheses that the broken linear trends as well as the trend itself are long-run excludable: the hypothesis was borderline rejected for the broken trend starting in 2001:1 and very largely rejected for the other broken trend and the trend.

Trying to identify the long-run structure underlying the two relations, the final interesting identified structure is given by the following normalized relations, accepted with a p-value of 0.438:

$$\begin{aligned}\hat{\beta}'_1 \tilde{x}_t + \hat{\delta}'_1 \Delta \tilde{x}_t &= p_t^{Pol} - p_t^{eu} - s_t^{Pol} + 79.551 \Delta p_t^{Pol} - 15.987 \Delta p_t^{eu} + 95.534 \Delta s_t^{Pol} + 0.005 t_{01:1} - 0.005 t + \dots \\ \hat{\beta}'_2 \tilde{x}_t + \hat{\delta}'_2 \Delta \tilde{x}_t &= p_t^{eu} + 0.167 s_t^{Pol} - 12.069 \Delta p_t^{Pol} + 2.411 \Delta p_t^{eu} - 14.454 \Delta s_t^{Pol} + 0.001 t_{08:2} - 0.006 t + \dots\end{aligned}$$

In the first relation we find significant evidence of 'strong form' of PPP, which is made stationary by the inflation differential between the two countries and by the changes in the exchange rate. The trend components show the same coefficients with opposite sign, as if the trend became insignificant from 2001:1 onward, thus implying a convergence between the relative prices and the nominal exchange rate when the inflations and movements in the exchange rate are taken into account.

The second relation doesn't have a direct economic interpretation, it's a relation between the level of prices in the euro-area and the nominal exchange rate.

The estimated adjustment coefficients - not reported for reasons of space coefficients - show that prices in Poland adjust significantly to both long-run equilibrium relations.

The I(2) common stochastic trend, which can be considered as the force driving this system of variables, results to be made up mainly by twice cumulated shocks to euro-area prices, and, in part, to the exchange rate.

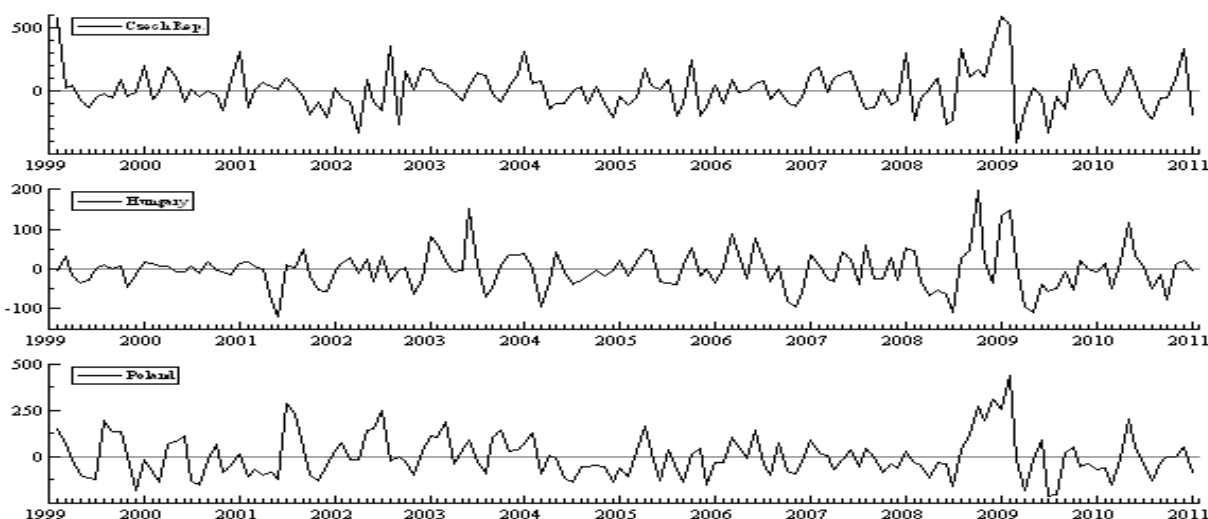
Conclusions

The paper deals with the problem of assessing stochastic convergence between time series as in Bernard and Durlauf (1995), but we consider also the possibility of structural changes occurring in the period of observation. We move from the work of Lee and Strazicich (2003), where they show that stochastic convergence is achieved when, controlling for at most two endogenously determined structural breaks in level and trend, the null hypothesis of a unit root in the log difference of the two series is rejected. According to them, rejection implies the conditional convergence of the series, or, in other terms, that the relation between the them is stationary, apart from a possible deterministic term. In other words, stationarity implies that there is evidence of a cointegration relation such that shocks to it have only temporary effects. The aim of the paper is to show that we can still detect stochastic converging series even if we fail to reject the null hypothesis of a unit root in the log difference of the two series. This is the case when the estimated relation between the series is a polynomial cointegration relation estimated within an I(2) model. This means that we cannot simply conclude that the series are diverging analyzing them within an I(1) model, but we should also consider the possibility of stochastic relative convergence between I(2) series, after controlling for possible structural breaks, as in Juselius (2006, chapters 16-18). This is particular useful when the observed time series show evidence of regime changes. Our empirical analysis refers to the catching-up process of the Czech Republic, Hungary and Poland towards the euro-area economy. The hypothesis is whether we can find

misspecification tests for the unrestricted VAR(2) model with dummies take the following values: the LM(1) test for first order autocorrelation is equal to 10.280 with a p-value of 0.328, while the LM(2) test for second order autocorrelation is equal to 11.326 with a p-value of 0.254; the LM tests show no ARCH effects, while the normality test shows some problems in the euro-area prices.

a stationary polynomial cointegrating relation involving the ‘strong form’ of PPP, which would imply a convergence between the relative prices and the nominal exchange rate. The structural changes occurring are the ones related to the recent economic crisis. We show that rejecting convergence in the I(1) model still allows to assess convergence in the I(2) model, where convergence is driven by an I(2) common stochastic trend. The results are interesting, in particular we find evidence of PPP in all the three countries (see Figure 1), when taking into account the inflation differentials and the changes in exchange rates. The signals of convergence result to be stronger for Hungary and Poland.

Figure 1: The graph of the polynomial cointegration relations representing the ‘strong form’ of PPP



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