Measuring Business Cycles for Pacific Island Countries

Lahari, Willie
National Statistical Office
Papua New Guinea and Solomon Islands
willie.lahari@otago.ac.nz

Understanding the dynamic behavior of business cycles enables policy makers in designing strategies aimed at mitigating the effects of these cycles or fluctuations.[117] Measuring business cycles among Pacific Island countries (PICs) is critical in determining stylised facts (e.g., whether business cycles among PICs are synchronised or if PICs have different business cycle phases etc.) that facilitates identification of appropriate policy response and strategies to counter the effects of aggregate macroeconomic fluctuations at the national and regional levels. In the Pacific Islands region, there is currently no specific evidence of business cycle stylised facts concerning PICs due, in part, to the lack of relevant, consistent and quality macroeconomic time series data (Lahari, et. al, 2011). Hence, there are currently no empirical studies undertaken on business cycles for the PICs apart from a recent (forthcoming) research by Lahari (2011).[118] This analysis makes an attempt to fill this void by measuring business cycles for six PICs namely Fiji, Papua New Guinea, Samoa, Solomon Islands, Tonga and Vanuatu. In addition, the degree of the business cycles for these PICs is assessed for similarities among the cycles.

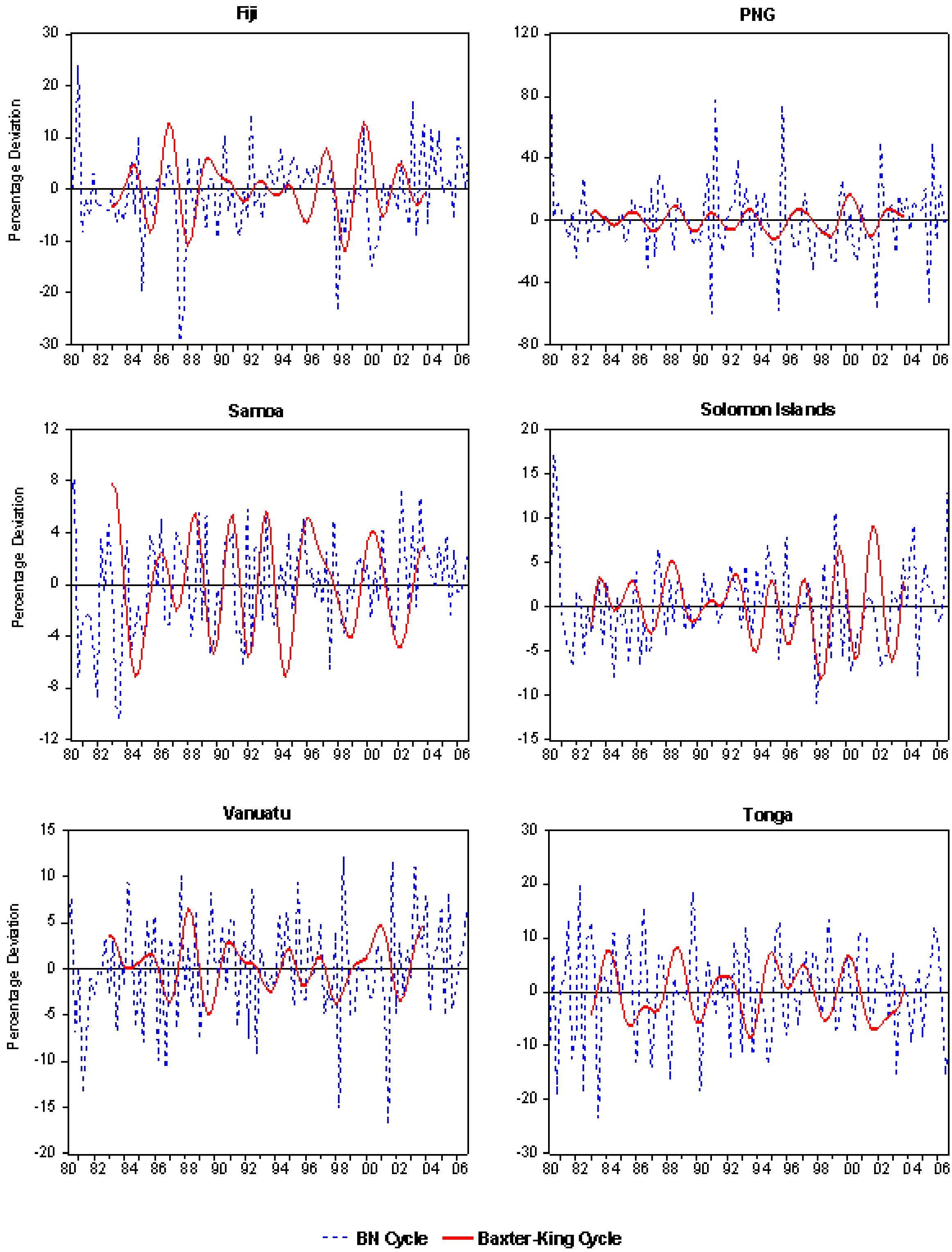
This analysis uses the newly constructed quarterly real GDP by Lahari et al. (2011) from 1980:1 to 2006:4. To ensure that the extracted business cycles are reliable and robust, comparing the outcomes from a number of filtering or decomposition methods are suggested (Canova, 1998). Two relevant methods are employed in this analysis. These include a non-parametric univariate filter, namely Baxter and King's (1999) band-pass filter and a parametric (model-based) decomposition procedure by Beveridge-Nelson (1981) following the state-space approach by Morley et al. (2003). In addition, an econometric testing methodology by Vahid and Engle (1993) is applied in assessing the extent of common business cycle synchronisation.

The choice of the Baxter-King filter was determined given its wider preference from a theoretical and conceptual basis, and considering the context for PICs. Analysis of the data for PICs demonstrates that the specified range for business cycles fits within the stated definition. Secondly, the choice of the Beveridge-Nelson (BN) (1981) univariate procedure provides another dimension to evaluating the business cycles. In contrast to the cycles generated by the Baxter-King filter that are defined within a specific period and whose cycles appear relatively smooth and persistent, the BN cycles are normally short and noisy (Morley et al. 2003). Thus in retrospect, the BN generated cycles for the PICs should reflect to some extent the behaviour of inconsistent growth phases among PICs that have been predominantly impacted by transitory shocks (e.g., frequent cyclones and volatility of world export commodity prices). In contract to the Baxter-King filter and the BN decomposition method, the testing method for common cycles by Vahid and Engle (1993) embraces the concept of the serial correlation common feature and cointegration and tests for common cycles among PICs. The technical details of the methodologies are illustrated in the Appendices.

The estimated business cycles generated by the Baxter-King filter and the BN decomposition methods are presented in Figure 1. The horizontal axis represents the quarters from 1980 to 2006. The vertical axis represents the percent of cyclical deviations. As expected, the BN cycles for the PICs appear generally noisy compared to the Baxter-King cycles. The latter cycles appeared smoother and more persistent. As stated earlier, this is attributed to the varying properties associated with the respective methods.

[117] See Burns and Mitchell (1946), Canova (1998), among others, for a discussion on the conceptual and methodological issues of business/growth cycles.
[118] See Lahari (2011).

Figure 1: Business Cycles for the PICs - Baxter-King Cycles and BN Cycles, 1980: Qtr 1 to 2006: Qtr 4



The common cycle test does not reject the null at the 5% significance levels. This indicates the presence of at least two co-feature vectors corresponding to two synchronised common cycles. The presence of the synchronised common cycles is an important precondition for a region-wide monetary policy framework such as in a monetary union. However, a single region or union-wide common policy to be effective depends on a single synchronised common business cycle among the PICs. Lastly but not least, this is a first attempt to measure business cycles for PICs and thus this work provides a useful contribution to new research and analysis of business cycles in Pacific Island region.

REFERENCES

Baxter, M. and King R.G. (1999). Measuring Business Cycles: approximate Band-Pass Filters for Economic Time Series. *Review of Economics and Statistics*, 81(4), 575-93.
Burns, A.F. and Mitchell, W.C. (Eds.) (1946). *Measuring Business Cycles.* New York: National Bureau of Economic Research.
Canova, F. (1998). Detrending and Business Cycle Facts, *Journal of Monetary Economics*, 41(3), 475–512.
Lahari, W., Haug, A. A., and Garces-Ozanne, A. (2011). Estimating Quarterly GDP for the South Pacific Island Nations, *Singapore Economic Review,* 56(1), 97-112.
Lahari, W. (2011). Business Cycle Synchronisation and Currency Union Among South Pacific Island Countries (forthcoming).
Morley, J.C., Nelson, C.R. and Zivot, E. (2003). Why are Beveridge-Nelson and Unobserved Component Decompositions of GDP so different? The Review of Economics and Statistics, 85(2), 235-243.
Vahid, F. and Engle, R.F. (1993). Common Trends and Common Cycles. Journal of Applied Econometrics, 8(4), 341-360.

APPENDIX 1: Baxter and King (1999) Filter

Baxter and King (1999) approximates the form of a 2-sided infinite moving-average, X_t, and derives a new series, X_t^{BK} in the following form,

$$X_t^{BK} = \sum_{k=-K}^{K} a_k X_{t-k} = a(L)X_t \qquad (A.1)$$

where L is the lag operator and symmetry is imposed on moving averages where $a_k = a_{-K}$ for $k = 1, \ldots, K$ to isolate trends in the series. The filtered stationary series, X_t^{BK}, in equation (A.1) is derived within the premise of the frequency-domain theory, in the form of the Cramer representation,

$$X_t = \int_{-\pi}^{\pi} \xi(\omega) d\omega \qquad (A.2)$$

where $\xi(\omega)$ represents the random periodic components that are mutually orthogonal $(E\xi(\omega_1)\xi(w_2)' = 0 \; for \; \omega_1 \neq \omega_2)$. Thus the filtered time series, X_t^{BK} in equation (A.1) can be represented as,

$$X_t^{BK} = \int_{-\pi}^{\pi} \alpha(\omega)\xi(\omega)d\omega \qquad (A.3)$$

where $\alpha(\omega) = \sum_{h=-K}^{K} a_h e^{-i\omega h}$ represents the frequency-response function of the liner filter. It is the weight attached to the periodic component $\xi(\omega)$. Thus from a frequency-domain logic, a band-pass filter, constructed from two low-pass filters, passes only frequencies $-\omega_1 \leq \omega \leq \omega_1$. It will have a frequency response function given by $\beta(\omega) = 1$ for $|\omega| \leq \omega_1$ and $\beta(\omega) = 0$ for $|\omega| > \omega_1$ within a time domain representation given by the two-sided infinite-order moving-average, $b(L) = \sum_{h=-\infty}^{\infty} b_h L^h$ with weights given by $b_h = \int_{-\pi}^{\pi} \beta(\omega) e^{i\omega h} d\omega$. The weights are simply $b_0 = \omega_1/\pi$ and $b_h = sin\,(h\omega_1/h\omega)$. To approximate the ideal filter with a finite *K*-th order moving average $a(L) = \sum_{h=-K}^{K} a_h L^h$ with frequency response function, $\alpha_K(\omega)$, the aim is to choose the weights of the approximating filter to minimize a quadratic loss function $Q = \int_{-\pi}^{\pi} |\beta(\omega) - \alpha_K(\omega)|^2 d\omega$ so as to minimise the discrepancies between the finite-sample filter and the ideal band-pass filter. An approximation to the ideal band-pass filter that passes frequencies between $-\omega_1 \leq \omega \leq \omega_2$ is then constructed with cut-off frequencies ω_1 and ω_2. In the case where stationary time series are required to be filtered from non-stationary series, Baxter and King show that the optimal approximate low-pass filter weights, $a_h = b_h + \theta$, can be adjusted by the adjustment coefficient θ where $\theta = (1 - \sum_{h-K}^{K} b_h)/(2K + 1)$. The weights for the optimal approximate band-pass filter are $a_h = (\bar{b}_h + \underline{b}_h) + (\bar{\theta} - \underline{\theta})$, where $\bar{\theta}$ and $\underline{\theta}$ are the adjustment coefficients for the upper and lower cut-off filters respectively. In order to implement the BK filter, the researcher must first choose the range of durations (periodicities) to pass through, that is, the cut-off frequencies, ω_1 and ω_2, and the order of the moving average, *K*.

APPENDIX 2: Beveridge-Nelson (1981) Decomposition

Beveridge-Nelson (BN) (1981)'s univariate decomposition, following from Morley et al.'s (2003) approach, begins with the theoretical equivalence of the following unobserved-components (UC) representation,

$$X_t = T_t + C_t, \qquad \text{(B.1)}$$
$$T_t = T_{t-1} + \mu + \xi_t, \qquad \xi_t \sim i.i.d.N(0, Q_\xi^2), \qquad \text{(B.2)}$$

where X_t is the observed series, T is the unobserved stochastic trend assumed to be a random walk with mean growth rate, μ, and C_t is the unobserved stationary cycle.[119] The trend innovation, ξ_t, is independently and identically distributed with mean zero and variance, Q_ξ^2. This process allows that C_t is a stationary and invertible ARMA(p, q) process whose cyclical innovations, ϵ_t, may be contemporaneously cross-correlated with trend innovations, ξ_t, where,

$$\phi_p(L)C_t = \theta_q(L)\epsilon_t, \qquad \epsilon \sim i.i.d\ N(0, Q_\epsilon^2) \qquad \text{(B.3)}$$
$$Cov(\xi_t, \epsilon_{t\pm k}) = \begin{cases} Q\epsilon\xi & \text{for } k = 0, \\ 0 & \text{otherwise} \end{cases}$$

The above covariance can take the value of zero or non-zero. Restricting the value of the covariance to zero may address the problem of correlation between the innovations of the stochastic trend and the cycle. To specify the necessary conditions, it was proposed that the autoregressive component (p) = *2,* as applied by Morley et al. (2003). This engages the cyclical process to be periodic. The UC-ARMA(p, q) model is then cast into state-space form. This process implies that the trend and cycle innovations are uncorrelated and that the model is augmented to include $Q_{\epsilon\xi} = 0$.[120] However, Morley et al. (2003) argue that differences in the implied trend and cycle in some data may often imply a non-zero covariance suggesting that the unobserved model may be misspecified. To resolve this, the conditions are specified in the context of the reduced-form ARIMA(p, d, q) since the UC-ARMA(p, q) implies an equivalent univariate ARIMA(p, d, q) representation. Substituting equations (B.2) and (B.3) into (B.1), we get the reduced-form ARIMA (p, d, q) where ΔX_t takes the form,

$$\phi_p(L)(1 - L)X_t = \phi_p(L)\mu + \phi_p(L)\xi_t + \theta_q(L)(1 - L)\epsilon_t \qquad \text{(B.4)}$$

where the right-hand side of equation (E2.4.) represent the sum of the MA(p) and MA($q^* + 1$) processes. This means that the UC-ARMA(p, q) and the reduced-form ARIMA(p, d, q) must have MA order $q^* = \max(p, q + 1)$. It implies that the autoregressive (AR) component of equation (E2.4) and the reduced-from ARIMA(p, d, q) are equivalent. Therefore the reduced-form ARIMA(p, d, q) must have $q + 2$ MA parameters and $p \geq q + 2$. Following from Morley et al. (2003) and earlier discussion by Morley (2002), the state-space approach is applied for computing the exact BN cycle given a reduced-form ARIMA(2,1,2) model. The state-space framework generalises that the reduced-

[119] The μ is allowed to behave as a random walk and in some cases with an addition of an irregular term. This change may have little influence on the estimated cycle (Morley et al. 2003).

[120] This set-up continues to be standard framework for the treatment of trend-cycle decomposition in the state-space framework (See Morley et al. 2003, among others).

form ARIMA(2,1,2) model when cast in state-space form is able to generate accurate forecasts since $\Delta X_t - \mu = [h_1, h_2 \dots h_k]X_t$, where $h_1, h_2 \dots h_k$ are the weight of the *i*-th element of X_t, in determining $(\Delta X_t - \mu)$. Thus $(\Delta X_t - \mu)$ is a liner combination of the first elements in the state-vector Z_t, given as,

$$Z_t = FZ_{t-1} + \nu_{t,} \qquad \nu_t \sim N(0, \psi) \tag{B.5}$$

where the eigenvalues of F take the values less than one in modulus. Thus, the BN trend is computed as,

$$T_t^{BN} = X_t + [h_1, h_2 \dots h_k]F(I-F)^{-1}Z_{t|t} \tag{B.6}$$

where the $Z_{t|t}$ term assumes the realised value of the state-vector since the elements are all observed at time t. The BN cycle is,

$$C_t^{BN} = -[h_1, h_2 \dots h_k]F(I-F)^{-1}Z_{t|t} \tag{B.7}$$

APPENDIX 3: Common Cycle Test

The common cycle testing approach (Vahid and Engle, 1993) is a test for the existence of cofeature combinations or common cycles. The test is preformed sequentially beginning with the null that there is at least one co-feature combination, s, against the alternative that there is none. If this does not hold, the null of at least two co-feature combinations are tested until the number of linear combinations is, $s \le N-r$; N represents the number of variables in the vector autoregressive model; r represents the of number of cointegrating vectors in the system. Thus the test statistic that determines the actual number of co-feature combinations, s, is presented as,

$$C(p,s) = -(T-p-1)\sum_{j=1}^{s} \ln\left(1-\ell_j^2\right)$$

where the ℓ_j^2, for $j=1,\dots,s$, are the s smallest estimated squared canonical correlations between ΔX_t and its related history $H(p) = (\Delta X_{t-1},\dots,\Delta X_{t-p}, e_{t-1})$ where e_{t-1} is the lagged error-correction term. T is the total number of observations of the variables of interest. Accordingly, when defining Z_t as the matrix of elements consisting of ΔX_t and H_{-1} as the matrix of elements consisting of $H(p)$, then $\tilde{\ell}^2, \dots, \tilde{\ell}_s^2$ are derived as the s smallest eigenvalues of $(Z'Z)^{-1}Z'\,H_{-1}(H_{-1}'H_{-1})^{-1}\,H_{-1}'Z$. This test statistic $C(p,s)$ under the null has an asymptotic χ^2 distribution with $s(Np+r)$ - $s(N-s)$ degrees of freedom, where p is the lag order of the vector error correction model.