A New VSI EWMA Average Loss Control Chart

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1. INTRODUCTION

When dealing with a variables quality characteristic, we normally employ one chart like $\overline{X}$, to monitor the process mean and another chart like R or S, to monitor the process spread. However it is more advantageous to use a single chart to achieve the same purpose. A single chart is simple to use and interpret; it would cut down the time, resource, manpower, money and effort. It would also detect the source of the changes – whether it is from the process mean or the process variability or both (see Cheng and Thaga[1]). In the last ten years, quite a bit of researches in this area have been done, such as the T chart (Chen, Cheng and Li[2]), the single B chart (Grabov and Ingman[3]), the Likelihood Ratio chart (Sullivan and Woodall[4]), the Max chart (Chen and Cheng[5]), the MSE chart (Cheng and Spiring[6]), and the weighted-loss chart (Wu and Tian[7]). Cheng and Thaga[1] gave an overview on the single charts developed.

In industry, quality of products and loss of productivities are important factors among competitive com panies. Loss funtion is widely used in industry to measure the loss due to the process variable deviates from the target value (Spiring and Yeung[8]). From Taguchi's philosophy (Gopalakrishnan, Jaraiedi, Iskander and Ahmad[9]), the target value is a vital process measurement. Sullivan[10] gave examples that stress the importance of monitoring the deviation from the target value. In practice, the in-control process mean may not be the target value. However, all the above papers of single charts assumed that the in-control process mean was equivalent to the target value.

A single control chart would be effective if it could detect the deviation from the target value with smallest loss. Furthermore, many papers on the performance of adaptive EWMA control charts had suggested using adaptive control schemes instead of the fixed control scheme. But, only a few researches proposed the double adaptive EWMA charts or a single adaptive EWMA chart to monitor the process mean and the variance simultaneously, like Chengular, Arnold and Reynolds[11], Reynolds and Stoumbos[12], Zhang and Wu[13], Costa and Magalh[14],Costa and Rahim[15], Yang and Chen[16], Yang and Yang[17], Yang and Yu[18], Wu, Wang and Wang[19] and Prajapati and Mahapatra[20]. So far, no single variable sampling interval (VSI) EWMA control chart detecting the deviation from process target value has ever been proposed. A single optimal VSI EWMA average loss chart is thus proposed to detect the deviation from the target value and improve the performance of the single average loss chart with fixed design parameters (FP). In section 2, a Taguchi loss function is introduced and the distribution of the average loss is derived. The FP and VSI EWMA average loss charts are constructed in section 3. Finally, the application of the optimal VSI EWMA average loss chart is illustrated by an example and the data analysis compares the performance among the VSI and FP EWMA average loss charts.

2. THE DISTRIBUTION OF TAGUCHI LOSS FUNCTION

Taguchi quadratic loss function is defined as

$$L = k' E(X-T)^2 ,$$

where L = loss, *k'* = coefficient of the loss, X = the quality variable of interest and T = the target value.
The average loss, AL, is then

$$\text{AL} = E(L) = kE(X-T)^2 . \qquad (1)$$

Let $X \sim N(\mu, \sigma^2)$ when process is in-control.

When μ and σ^2 are unknown, the average loss (AL) is estimated by

$$\hat{AL} = k\sum_{i=1}^{n}(X_i - T)^2/n \tag{2}$$

where n = the sample size.

Without lose generalization, let k=1. $\hat{AL}$ can be rewritten as

$$\hat{AL} = \frac{n-1}{n}S^2 + (\bar{X} - T)^2 \tag{3}$$

Under normality assumption, S^2 and $\bar{X}$ are independent, the in-control distribution of the statistic $\hat{AL}$ is derived as

$$\hat{AL} \sim \frac{\sigma^2 \chi^2_{n,\tau_0}}{n} \tag{4}$$

where χ^2_{n,τ_0} is a non-central chi-square distribution with n degrees of freedom and non-centrality $\tau_0 = n\delta_3^2$.

Let the in-control mean shift right and variance increase, $X \sim N\left(\mu + \delta_1\sigma, \delta_2^2\sigma^2\right), \delta_1 > 0$ and $\delta_2 > 1$, when the process is out-of-control. The out-of-control distribution of $\hat{AL}$ is derived as

$$\hat{AL} \sim \frac{\delta_2^2\sigma^2\chi^2_{n,\tau_1}}{n}, \tag{5}$$

where non-centrality $\tau_1 = n[(\mu + \delta_1\sigma - T)/\delta_2\sigma]^2 \;\; = n[(\delta_1 + \delta_3)/\delta_2]^2$.

3. THE FP AND VSI EWMA AVERAGE LOSS CONTROL CHART

Since the $\hat{AL}$ consists two components - one related to the sample variance (S^2) and the difference of the sample mean ($\bar{X}$) and the target (T). Either one of the in-control variance had increased and/or mean had shifted right, the statistic $\hat{AL}$ gets larger. Hence, using the distribution of the statistic $\hat{AL}$ can construct EWMA chart to monitor the deviation from the process target value. Alternatively, the constructed EWMA chart is equivalent to detect the shifts in the process mean and the variance since $\hat{AL}$ is the linear combination of the sample variance and the difference of the sample mean and the target value. Consequently, an out-of-control sample would easily exceed the upper control limit of the EWMA $\hat{AL}$ chart. It is reasonable to set the lower control limit to 0. Let a FP EWMA $\hat{AL}$ chart have fixed sample size (n), fixed sampling interval (t_0) and a fixed control factor (k) of upper control limit (UCL) or a fixed false alarm rate (α). First, we define EWMA monitoring statistics

$$EWMA_{\hat{AL}_i} = \lambda \hat{AL}_i + (1-\lambda)EWMA_{\hat{AL}_{i-1}}, \quad 0 < \lambda \le 1,$$

where $\hat{AL}_i$ represents the ith sequentially recorded average loss from the process.

The mean and standard deviation of $EWMA_{\hat{AL}_i}$ are

$$E(EWMA_{\hat{AL}_i}) = (n + \tau_0) \text{ and } \sqrt{V(EWMA_{\hat{AL}_i})} = \sqrt{\frac{2\lambda(1-(1-\lambda)^{2i})(n+2\tau_0)}{2-\lambda}}.$$

If time is infinite then $\sqrt{V(EWMA_{\hat{AL}_i})} = \sqrt{\frac{2\lambda(n+2\tau_0)}{2-\lambda}}$.

The control limits for the EWMA Chart are usually based on the asymptotic standard deviation of the control statistic. Hence the constructed upper control limit (UCL) and the lower control limit (LCL) of the FP EWMA $\hat{AL}$ chart are as follows

$$UCL = (n+\tau_0) + k\sqrt{\frac{2\lambda(n+2\tau_0)}{2-\lambda}} \qquad LCL = 0$$

and plot $EWMA_{\hat{AL}_i}$ on the chart. If any $EWMA_{\hat{AL}_i}$ > UCL, the process is deemed to be out of control. The chart parameters, k and λ, are chosen that they would satisfy certain in-control average time to signal (ATS_0) requirements. ATS_0 is defined as the average time from the start of an in-control process until a false signal is obtained from a chart.

To quickly detect the increases in the difference of the process mean and the target and the variance simultaneously, a VSI EWMA $\hat{AL}$ chart is designed. The VSI EWMA $\hat{AL}$ Chart is as follows

$$UCL = (n+\tau_0) + k\sqrt{\frac{2\lambda(n+2\tau_0)}{2-\lambda}}$$

$$WL = (n+\tau_0) + w\sqrt{\frac{2\lambda(n+2\tau_0)}{2-\lambda}}$$

$$LCL = 0$$

The region between LCL and WL is called the 'central region' (CR), that between WL and UCL the 'warning region' (WR) and that above UCL the 'action region' (AR). Two variable sampling intervals, (t_1, t_2), are adopted. The long sampling interval (t_1) is adopted when the plotted point fell into the CR; the short sampling interval (t_2) is adopted when the plotted point fell into the WR, where $0 < t_2 < t_1 < \infty$.

When $t_1 = t_2 = t_0$ and $w = 0$ the VSI EWMA $\hat{AL}$ chart is reduced to a FP EWMA $\hat{AL}$ chart.

Assuming that the process is in-control at the beginning, the first sampling interval (t_q) is randomly taken from the process. During the in-control period all samples including the first one, should have a probability of p_{01} to adopt (t_1) and a probability of p_{02} to adopt (t_2), where $p_{01} + p_{02} = 1$ and

$$p_{01} = P\left(\hat{AL} < w\sigma^2 \mid \hat{AL} < k\sigma^2, \delta_1 = 0, \delta_2 = 1\right) = \frac{F_{\chi^2_{n,\tau_0}}(nw)}{F_{\chi^2_{n,\tau_0}}(nk)}, \tag{6}$$

where $F_{\chi^2_{n,\tau_0}}(\bullet) =$ the cumulative probability function of a non-central chi-square distribution with n degree of freedom and non-central parameter $\tau_0 = n\delta_3^2$.

$p_{02} = 1 - p_{01}$.

To evaluate the performance of the FP, the VSI EWMA $\hat{AL}$ charts and the joint $\overline{X}$ and S charts, the adaptive control schemes should be compared under equal conditions; that is, they should demand the same ATS_0 under the in-control process. Under the same ATS_0, ATS_1 evaluates the performance of a chart. ATS_1 denotes the out-of-control ATS or the average time of the out-of-control process. The smaller ATS_1 indicates the better performance of a chart.

Following Lucas and Saccucci [22], the *ATSs* of the New chart are evaluated by Markov chain approach.

3. AN EXAMPLE

Consider a data set from Braverman (1981). Presently, the FP EWMA $\hat{AL}$ chart is used to monitor the difference of the process mean and the target and the variance every hour ($t_0 = 1$). When the control chart indicates that the process is out-of-control, it requires adjustment. To construct the control chart, 28 samples of size $n = 4$ are taken. The estimated in-control process mean and standard deviation are $\hat{\mu}=9.73$ and $\hat{\sigma}^2$ =48.42 respectively. The target value of X is 12, T = 12. Consequently, δ_3 = 0.326. From historical data, the estimated failure frequency of the machine is 0.1 times per hour (or $\gamma = 0.1$). The failure machine would increase the mean and variance to $N(\mu+\delta_1\sigma, \delta_2^2\sigma^2)$ with δ_1 = 0.5 and δ_2 = 1.1. The FP EWMA $\hat{AL}$ chart has λ = 0.2, control limits UCL = 200.46 and LCL = 0 with in-control ATS_0 = 370. The ATS_1 of the FP EWMA $\hat{AL}$ chart is 10.73 h. The slowness with which the FP EWMA $\hat{AL}$ chart detecting shifts in the process (δ_1 = 0.5 and δ_2 = 1.1) has led the quality manager to proposing the construction of the $\hat{AL}$ chart with optimal variable sampling intervals (VSIs).

Following the guidelines in Section 2, to construct the optimal VSI $\hat{AL}$ chart we set $t_L = 0.1$ and $t_U = 2$. Using Fortran IMSL BCONF subroutine to minimize ATS_1 under the constraints t_L =0.1< t_2 < t_0 < t_1 < $t_U = 2$, the determined optimal VSIs are (t_1 = 1.92, t_2 = 0.1), and the minimum ATS_1 = 2.5 h. Hence, the optimum VSI $\hat{AL}$ chart is constructed:

UCL = 7.77 UWL = 4.23 LCL = 0.

The VSI scheme improves the sensitivity of the FP EWMA $\hat{AL}$ chart. From the example above, the optimal ATS_1 of the VSI EWMA $\hat{AL}$ chart has been reduced from 10.73 h to only 2.50 h, representing a saving time of 76.70%. Hence, the optimal VSI EWMA $\hat{AL}$ chart outperforms.

The strategy of the process monitoring is

(1) The variable sampling intervals, (t_1 = 1.92) or (t_2 = 0.1), is determined randomly when the process and starts the sample of size four is taken.

(2) When the statistic $EWMA_{\hat{AL}}$ fell into CR, the long sampling interval (t_1 = 1.92), is adopted next time. When the statistic $EWMA_{\hat{AL}}$ fell into WR, the short sampling interval (t_2 = 0.1) is adopted next time.

(3) When the statistic $EWMA_{\hat{AL}}$ fell into AR, the process stops to search and discard the assignable cause(s) and bring the process back into control.

Figure 1 shows the positions of the sample statistics on the optimal VSI $EWMA_{\hat{AL}}$ chart for the twenty-eight sample statistics. All the statistics fell in the chart, the process looks in control. The phase I chart is used to monitor the new process samples, the sample number 29-38, from the process (Table 1). The process indicates out of control from sample 31-38 (Fig. 1). The optimal VSI $EWMA_{\hat{AL}_i}$ chart performs quite well.

4. CONCLUSION

The proposed single VSI EWMA chart to simultaneously monitor the increase in the difference of the mean and the target and the variance. It improves the performance of the FP EWMA chart by increasing the detection speed. We found that the optimum VSI EWMA $\hat{AL}$ chart always works better (in the cases examined) than the FP EWMA $\hat{AL}$ chart.

We recommend to use the optimum VSI scheme in monitoring a process when quality engineers were unable to specify the VSIs. This paper proposed a single chart for a single process. However, the studies of the variables control scheme could be extended to study CUSUM/EWMA charts for the multiple steps of cascade processes.

Table 1. The new samples on the optimal VSI $EWMA_{\hat{AL}_i}$ chart

No.	$EWMA_{\hat{AL}}$	Which region of the chart?	No.	$EWMA_{\hat{AL}}$	Which region of the chart?
29	6.51	WR	34	14.76	AR
30	7.18	WR	35	17.23	AR
31	10.15	AR	36	16.51	AR
32	11.51	AR	37	17.80	AR
33	12.56	AR	38	20.51	AR

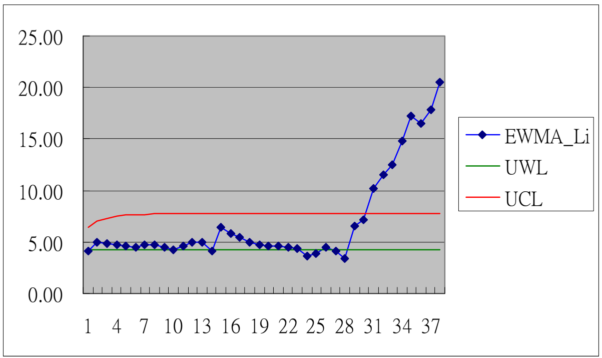

Figure 1. All sample statistics on the optimal VSI $EWMA_{\hat{AL}_i}$ chart

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