Continued Statistics Teachers formation at Colegio de Ciencias y Humanidades, Mexico

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INTRODUCTION

In Mexico there is a wide range of institutions of higher education, including two public universities: the National Autonomous University of Mexico (UNAM) and the National Polytechnic Institute (IPN). The UNAM has two different subsystems, one of which is the Colegio de Ciencias y Humanidades (CCH), whose philosophy is that students should be self-critical and responsible for their own learning as individuals. It offers a more advanced way of education.

One of the innovations proposed by the CCH relates to students studying statistics. A student entering the third and final year can choose one or two subjects related to mathematics from among three options: differential and integral calculus, cybernetics and computer science, and statistics and probability. Those students choosing statistics will face a course whose topics range from descriptive statistics to confidence intervals and hypothesis testing (for mean and proportion).

Statistical inference topics are usually considered too advanced for high school students, but the experience of 40 years has shown CCH that at a basic level it can be understood by pupils but only if their teachers have a good background in both statistics and in teaching statistics.

Most in-service statistics teachers have not studied education to any great extent but are mathematicians, engineers, and actuaries. Nicholson, Road and Darton (2003), Batanero, Godino and Roa (2004), Pecky and Gould (2005) and Conteras, Batanero, Diaz and Fernandes (2011) claim that often mathematics teachers have some basic training in probability and statistics, but generally have not been trained to teach the subjects. Most of them need pedagogical instruction based on concepts and practice.

According to Lopes (2003), a proposal to work with teaching of statistics related aspects would allow teachers to live experiments, and analyse samples in the same modelling processes that they will develop with their students. Several studies with this objective have been conducted, such as those of Cordani (2002), Watson (2006), Peck and Gould (2005), Amorim (2006), Gattuso and Pannone (2006), Innabi (2006), Contreras, Batanero, Diaz and Fernandez (2011), and Meletiou and Paparistodemou (2011).

Accordingly, the Colegio de Ciencias y Humanidades (CCH, Mexico), as an education institution interested in teacher training, offers 20-hour or 40-hour courses twice a year. Some of these courses are specifically designed for statistics and conducted by the two first authors of this paper; didactical sequences are shown, and teachers are asked to solve them in order to experience them from a student´s perspective, at the same time understanding how the sequences may work didactically.

These courses are divided into three stages. In the first stage, the teacher solves the sequence in the physical-experimental environment (paper and pencil). In the second stage the instructor describes the theory behind the sequence, discusses it with teachers, and at the same time shows where in the sequence the theory applies. The third stage in most of the sequences is designed to be developed by a software tool. It is considered that the course will be more significant if they can see theory and also experience the sequence.

In this context, this work aims to show the didactical sequence in relation to probability concepts, 'Monica's random walks' (MRW), as an example of the course activities offered by CCH for statistics teachers.

COURSE STAGES

First Stage

The didactical sequence 'Monica's random walks' allows us to work with probability concepts such as events, sample space, single event probability, to explore the difference between random and deterministic experiments, to approach probability from relative frequencies, to calculate theoretical probability from a tree diagram, and to compare observed and expected patterns.

This activity was devised by Fernandez and Fernandez (1999) for teaching binomial distribution to students and was adapted by Cazorla and Santana (2006) for use in elementary schools.

In the first stage the teachers work on the didactical sequence. The full activity has four different sections and 23 questions. In each section, questions must be answered on the basis of a previous action. In the first section, students have to read this story:

> 'Monica (the girl on the lower left corner) and her friends live in the same quarter. Monica´s house is four blocks away from the houses of Horácio (dinosaur), Cebolinha (three-haired child), Magali (girl in centre), Cascão (the child with braces) and Bidu (dog), according to Figure 1. Monica used to visit her friends on weekdays in a pre-set order: Monday, Horácio; Tuesday, Cebolinha; Wednesday, Magali; Thursday, Cascão; and Friday, Bidu. To make those visits more exciting, the group decided that the friend to be visited by Monica should be chosen at random. Therefore, when leaving home and on each crossing, Monica has to toss a coin; when the head occurs (H), she will walk a block north, and when the tail occurs (T), she will walk a block east. Each toss represents a block on her route. Monica has to toss the coin four times in order to reach a friend's house'.

After the teachers have read the story, they answer some questions without tossing the coin: What is the difference between Monica's former visit to her friends and its new shape? What are the possible outcomes when flipping a coin, What is the chance to get a face and to get a tail? Do all friends have the same chance of being visited? For all questions students must justify their answers.

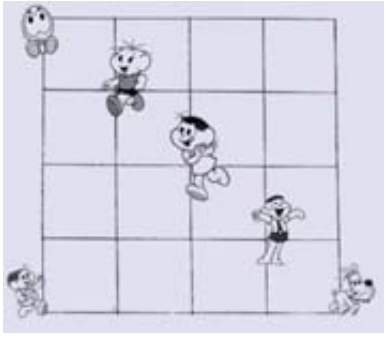

Figure 1. Map of Monica's random walks

In section II, teachers carry out an experiment by tossing four coins thirty times: heads mean Monica should go to north and tails east. They then answer the following set of questions: 'Who is more likely to be visited, Magali or Horace? Is there a chance of Monica not visiting a friend? After carrying out the experiment, would you change your answer to the question of whether all friends have the same chance of being visited? Systematize the results in the table of frequency distribution (TDF). Once you have organized the results in the TDF, would you change your answer to the question of whether all friends have the same chance of being visited? You are will be given a transparent sheet with two spaces to build charts, and pens. In the grid above represent the data of relative frequency given in Table 1. Compare results with those of your colleagues. Are they the same?'.

In section III, teachers have to construct a tree diagram, writing the obtained sequence, the number of heads and the friend visited, and answer the following questions: ' How many ways are there altogether? Find out if there is a common link to all paths leading to each of the five friends. Once you have analysed how many roads lead Monica to the house of every friend, would you change your answer to the question of whether all friends have the same chance of being visited? Analysing and systematizing the results of the tree diagram, fill the table with the number of paths and the probability of each friend'.

In section IV, they have to compare theoretical and frequency probabilities, and the random and deterministic experiments, answering the following questions: 'What is the difference between these two ways of assigning probabilities? Analysing the results, can you say which of these two ways of assigning probabilities is more appropriate? On the transparency, at the bottom space, represent data from the constant probability in Table 3. Compare results with those of your colleagues. Are they the same? Do you think the NEW probability distribution of Monica's visits to her friends is fair? If you think this distribution is unfair, could you indicate another way to choose the friend to be visited by Monica?'.

Second Stage

In the second stage a discussion about theories can be carried out in two ways: by using articles relating theory to sequence; the second by relating sequence's objectives to theory's main characteristics. In the case of MRW, three theories can be noted: Didactic Situations (Brousseau, 1996), Anthropology of Didactics (Chevallard, 1992) and Ontosemiotic theory (Godino, 2002)

To discuss the didactic situations theory (Brousseau, 1996), it may be presented the main features and its relations to the purposes of MRW. According to Maia (2007) this theory aims to 'study the phenomena that interfere with teaching and learning process, proposing a theoretical model to construct, analyze and experiment didactic situations, taking into account the interactions between teacher and student, mediated by knowledge in a teaching situation." For Brousseau (1996) problems should be chosen in such a way that the student can act, speak, think and evolve on their own, and the teacher can institutionalize the concepts at the end of the activity.

Maia (2007) further clarifies that:

> [...] between the time the student accepts the problem and produces his response the teacher does not interfere or suggest the content that he intends to be learned, the student acquires new knowledge fully justified by the internal logic of the situation and it can be built without didactic reasons. The author names this situation adidactical. Now, the didactic situation is characterized as a set of interactions between the teacher and the proposed problem, which goal is to learn, since the teacher returns to the student an adidactic situation.

The application of MRW with students fits the characteristics of both a\adidactic and didactic situations, which is why this theory is discussed with teachers.

The theory of the anthropology of didactics (TAD) is described by Nagamine, Henriques and Cazorla (2010). According to these authors, the TAD is a model for institutional analysis of human action, consisting of four concepts: Task, Technique, Technology and Theory. Task (T) contains at least one subtask t; Technique (τ) is how to accomplish a task type T. Technology (θ) is the rational discourse which aims to

justify and clarify the use of Technique τ, and enable the achievement of tasks of type T. The fourth and last concept is called Theory, represented by Θ, and serves to explain and make understandable a Technology θ. The four concepts represent a complete praxeological organization, [T / τ / θ / Θ], which can be decomposed into two blocks [T / τ] and [θ / Θ], constituting, respectively, the know-how {praxis} and the technological-theoretical environment [logos].

Nagamine, Henriques, Cazorla (2010) presented an a priori analysis of the first two sessions of MRW, which has a task (T), consisting of a sequence of subtasks (t), which can be performed using various techniques (τ) justified by the technology (θ) that uses the theory (Θ) of Probability as an object of study. For example, in sub-task (t): 'Do all friends have the same chance of being visited?'. The goal is to see how the subject is imbued with the concept of equal likelihood (τ), i.e. if there are five friends, then the chance is one in five (1 / 5). If this is the answer, it means that the person does not realize that the paths are equally likely, but not the friends, since they have a different number of paths leading Monica to visit them. Now, technology (θ) consists of the possible paths resulting from tossing the coin four times.

These authors concluded that, it is possible to identify conflicts when asking for some tasks, allowing for corrections and improvement of MRW. This is an important aspect, since it is assumed that all students are able to read and respond to tasks without difficulty, not requiring the teacher's help, but just following the didactic situations theory.

The ontosemiotic theory can be discussed with teachers with reference to the article by Gusmão and Cazorla (2009), who applied the MRW with teachers of mathematics This theory is concerned with the problem of the meaning of mathematical objects, both on a personal level (for example, manifested by the student) and institutional (for example, manifested by the teacher). It studies the phenomena of scholar didactic transposition trying to integrate ontological, syntactic, semantic, pragmatic, and sociocultural aspects (Gusmão and Cajaraville, 2007).

A semiotic analysis technique allows the characterization of the institutional and personal meanings expressed during mathematical activity and, therefore, thorough analysis of the semiotic conflicts derived from solving concrete mathematical problems. According to Godino, the semiotic conflicts:

> [...] refer to any disparity or discrepancy between the meanings assigned to the same expression for two subjects (persons or institutions) in communicative interaction and may explain the difficulties and limitations of implemented teaching and learning (2002, p. 246).

To identify potential semiotic conflicts in this approach six elements must be taken into account that highlight the typology of objects: language, situations, procedures, concepts, propositions and arguments.

Gusmão and Cazorla (2009), analysing the MRW, found that in the activity development the student must:

> [...] use a determined verbal language (a coin toss, random, etc.) and symbolic ("C to Head", "X to Tail," CCCX etc.). This language is ostensibly part of a series of *concepts* (sample space, events, probability, etc.), *propositions* (equiprobable events are equally likely) and *procedures* (repeatedly throw coins and note results, etc.) that will be used in the preparation of arguments to decide whether the actions that comprise the practice are satisfactory (p.3).

Gusmão and Cazorla (2009) concluded that MRW was feasible for teaching basic concepts of probability, but pointed out the presence of several semiotic conflicts, mainly because of the poor prior knowledge of teachers, who were experiencing some of these concepts for the first time.

Third Stage

In the third stage, the process required simulation with Fathom, and although some teachers were already familiar with this software, it was shown to the group to convey the basic idea behind its operation. Subsequently they were asked to perform the simulation of the experiment 1000 times in the way they deemed best. At first there was a proposal to use the command RandomPick, but they immediately realized that this would mean friends' equiprobability. Essentially there were three proposals: the first was to

randomly generate zero or one to four different coins and then calculate the sum of the four outcomes; the second was to use the command RandomBinomial; the third was to enter a database of two values (Heads and Tails), take repeated samples of size four, with replacement, and measure in each the number of Heads.

They also have to determine the shape of distribution of the visits to each friend using graphs and compare results from the 30 coin-tossings in the physical-experimental and theoretical probability tests with the results of their Fathom simulation.

In all simulations the outcomes were as expected for the experiment, and it was noted that computer simulation is a good teaching tool for supporting the MRW sequence.

FINAL CONSIDERATIONS

Given high theoretical level for high school that the training in statistics that the CCH is looking for its students, it is clear that the training of teachers in the subject should be better, too, in terms of both subject and teaching skills. Inference topics require good knowledge of the elements of probability, so support for the training of teachers in these areas is important. In that context, the MRW sequence shows itself not only to be an alternative for student learning, as already mentioned, but also an alternative for teachers' training which allows them to experience the strategy from the student perspective, in addition to experiencing different educational theories in practice and in a very specific case.

In addition, we consider that the activity can actually contribute to the teaching of basic concepts of probability and therefore help students to improve their level of probabilistic literacy.

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ABSTRACT

In Mexico, most in-service statistics teachers have not studied education; they are mathematicians, engineers and actuaries, so most of them need pedagogical instruction based on concepts and practice. The Colegio de Ciencias y Humanidades (CCH, Mexico), as an educative institution concerned about teacher training, offers 20-hours or 40-hour courses twice a year. In one of these courses for statistics, led by the two first authors in this paper, didactical sequences are shown, and teachers are asked to solve them, in order to experience the sequence from a student´s perspective, at the same time realizing how that sequence may work didactically. These courses are divided into three stages. In this context, this work aims to show the didactical sequence for working with probability concepts, 'Monica's random walks' (MRW) by Cazorla and Santana, as an example of the activities provided in the course. In the first stage of the course the sequence is based on Brousseau's theory of didactic situations and has been studied by researchers together with Godino's ontosemiotic theory and Chevellard's anthropology of didactics theory. In the second stage, the teachers solve four different sections and 23 questions. In the third stage, teachers with some knowledge of the Fathom software are asked to do 1000 times the experiment for MRW. The results obtained encourage us to believe that this proposal for continuing teacher training at CCH may help teachers with conceptual and didactical material, and update them in terms of educational theories as well.