Realized volatility models for electricity markets

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Introduction

Electricity is a *new* commodity with special features. Its risk properties and the dynamics of spot price variability are of primary concern for investors and operators. Therefore the volatility structure of electricity prices is analyzed, modeled and the forecasting ability is examined given the important applications in risk management and derivative pricing. In this paper, the volatility of Italian electricity prices is explored making use of the available intra–daily information to provide empirical evidence on volatility persistence and effects of explanatory variables. Previous studies on electricity volatility focussed on several aspects. Recently, [2] show that increasing the number of participants induces less volatility. A positive relations between liquidity, volume (meant as trading activities) and volatility of future prices was found by [10]. However, their approach is not feasible with Italian data[1]. We instead are going to consider and to test the impact of volume and realized intr-daily volatility on price volatility. Therefore, this paper will contribute to the first estimation of the relationships between intra–daily information and volatility in the Italian spot market making use of hourly day–ahead prices and traded volumes. We characterize the dynamics of electricity spot volatility in an ARMA–GARCH framework using intra-daily information, firstly the well known "realized volatility" and secondly the demanded volumes. Then we perform volatility forecasting on zonal basis to verify if these explanatory variables improve the forecasting performance. We show that the realized volatility explains well the conditional variance and reduces its persistence, hence inducing better volatility forecasts. We implement our analysis using hourly prices and volumes of the Italian electricity wholesale market, considering two zones in which the Italian market is split. Other authors have considered the intra–daily information, as in [4], who propose a model to distinguish between jump and non-jump components of the total variation to the aim of characterize and forecast the volatility of electricity prices. Instead, [12] propose the realized variance as ex–post measure for volatility, defined as the sum of the squared intra-day returns, for forward contracts (one quarter and one year ahead prices) to evaluate market volatility and to examine if the forecasting performance is improved by explanatory variables. They also try to explain this observable volatility with other correlated variables, as trading volume, time–to–maturity, asymmetries for negative shocks, and weekly seasonality. [13] use high frequency data from Australian markets to model volatility using ARCH-type models together with contemporaneous demand volumes, half–hour of the day, day of the week and

[1]According to the Italian independent system operator, liquidity is the ratio of volumes traded on the exchange (day–ahead market) to total volumes including bilateral contracts. In other words, it is the ratio of power traded on the exchange to total power volumes traded over the counter. By definition, even if hourly liquidity, volumes and prices are provided, the determination of liquidity strictly depends on volumes, hence it is normal to observe dependance (or strong correlation).

month of the year, but they are concerned about the moment in which intra–day information arrives and not in using all available high frequency information, as we propose. Moreover they document persistent volatility processes and a positive volume–volatility relationship, whereas we prove, with lagged volumes, that the relation is negative and more importantly that adding more information reduces the persistence of market volatility, inducing better volatility forecasts.

The paper is structured as follows. The following section is devoted to description and preliminary analysis of the employed data. Then proposed models are formulated and estimated on the Italian market. Finally, some conclusions and remarks are presented in the final section.

Data description

The dataset used in this study concerns Italian hourly electricity prices and purchased volumes from January 1, 2008 until December 31, 2010. The dataset is compiled by Italian independent system operator, *Gestore del Mercato Elettrico (GME)*, and there are no missing observations. The Italian electricity market has been divided in seven physical zones [2] taking into account production and market trading mechanisms (for a detailed explanation see [9]). In the present study we have deliberately excluded two zones (South and Central South) because Calabria was eliminated at the end of 2008, and this structural break has caused a strong level shift in both series of volumes and prices of the two cited zones directly afterwards. This hourly dataset has been used to construct three series: the first two are daily prices and realized daily variances of hourly returns, the third one is given by cumulated daily volumes. Daily prices were computed as arithmetic means of 24 hourly prices determined each day; then logarithmic hourly returns were computed and used to determine the intra–daily variance as measure of realized volatility. And finally, the total daily demand was determined by adding all hourly quantities of electricity demanded on one day. Volume and price dynamics at national level[3] for year 2008 are reported in Figure 1 where the upper panels show the daily series, whereas the lower panels show the weekly structure of both variables. The average values are computed along the whole observed period for each day of the week[4]. Two features can be observed and they reflect different forms of seasonality of electricity prices: the first one is a monthly behavior with higher demand of electricity during winter and summer with corresponding higher prices; the second one is a clear intra–week pattern with lower demanded volumes over the week–end and corresponding lower prices.

Instead of analyzing prices, we concentrate our attention on returns. The analysis of electricity returns is more important than the analysis of prices when the focus is on the volatility. Indeed, the analysis of returns gives insights about the process of instantaneous growth of prices as they measure the time–to–time variations of prices, and indeed returns are very carefully monitored by investors operating on electricity markets since volatility is of extreme interest in risk management.

GARCH models with realized volatility

The well–known stylized facts of electricity prices (see [14]) can be well modeled within the framework of ARIMA–GARCH processes. The first part of the process allows to capture empirical features of the conditional mean, while the GARCH component well fits the volatility of prices by the estimated conditional variance of the process. In the literature about electricity prices at least two approaches have been followed to take into account market volatility: the first one refers to the analysis of electricity prices p_t as in [11], [14] and [15], among others; whereas the second one considers

[2] These are North, Central North, Central South, South, Calabria, Sicily and Sardinia.

[3] The physical zones show similar features and are not reported for lack of space.

[4] Then for instance, the average demanded volumes on Monday are obtained as the arithmetic mean of the volumes purchased on Monday from 01/01/2008 to 12/31/2010.

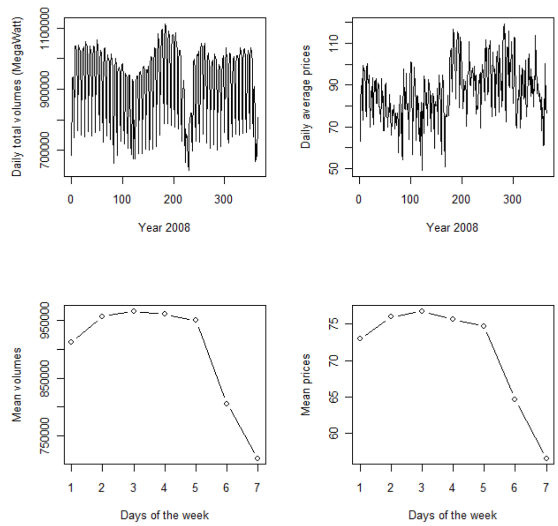

Figure 1: National demanded Volumes and PUN Prices. Upper panels show the time dynamics of variables in 2008. Lower panels report the average values of each day of the week computed on the whole data–set. On the horizontal axis of lower panels *1* is for Monday and *7* for Sunday.

analysis of electricity price variations computed as log-returns or simple returns, as in [3], [17].

Among first studies, [9] performed an analysis of the Italian zonal market using daily medians of prices and standard deviations and implemented ARFIMA models to account for a long memory autocorrelation structure. Moreover, [8] used asymmetric GARCH models on returns of daily average prices to study the volatility dynamics of European markets.

Given the strong seasonal behavior highlighted previously, a seasonal adjustment has been taken into account when estimating the proposed models which can be formalized as follows:

$$r_t = c + \phi_1 r_{t-1} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \gamma_1 Vol_{t-1} + \nu_1 D_1 + \ldots + \nu_6 D_6 + \nu_7 CE + \varepsilon_t \tag{1}$$

with $\varepsilon_t | I_{t-1} \sim NID(0, \sigma_t^2)$ and

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \tau_1 Vol_{t-1} + \tau_2 RV_{t-1} \tag{2}$$

for $t = 1, ..., T$, $r_t = log(P_t/Pt - 1)$ is the return on electricity price (P_t) at time t. The $\theta_j \varepsilon_{t-j}$ terms represent the moving average component of the return dynamics with coefficients θ_j for $j = 1, \ldots, q$. D_j with $j = 1, \ldots, 6$ are dummies for days of the week and ν_j are the corresponding coefficients; CE is a dummy variable accounting for calender effects and ν_7 is the corresponding coefficient; the $\phi_1 r_{t-1}$ term represents the autoregressive component of the return dynamics. Finally, Vol_t and RV_t are, respectively, the total volumes of electricity dispatched at day t and the daily realized volatility computed as the variance of the hourly returns on prices during day t. Thus, the coefficients τ_1 and τ_2 estimate the effect that purchased volumes [5] and realized volatility exert on the conditional volatility. To avoid problems of hendogeneity we included lagged values of volumes and realized volatility.

Usually GARCH models are estimated using daily time series of financial returns. The main goal of this class of models is to use information about the current level of volatility to forecast future levels of volatility. The most common measure of instant volatility for financial time series is given

[5] Remember that volumes are measured in MW and their levels are far larger than the average values of price volatility. Then, to avoid values of estimates not easy to read, we decided to express volumes in hundred thousands (100,000) of MW.

by the squared value of returns. However a single daily return contains limited information about the current level of volatility, so that GARCH models are quite slow in capturing the quick volatility changes and they might take many periods to reach the new level (see [1]). In this paper we exploit the common frequency of electricity prices which are usually determined on hourly base (for most of the European markets) or on half–hourly base (for instance, in UK). The intra–daily frequency allow to compute realized measures of volatility such as RV_t or proxies of realized volatility, such as Vol_t. We included both in the conditional variance equation and we observed less persistence since this new intra–daily information makes the β coefficient non significant. Thus the final model for the conditional variance is a ARCH(1) model with regressors. Moreover, following [7] and [16] we expect better estimators and improved forecasts with this high-frequency data.

Empirical analysis: the Italian market

High-frequency data should give rise to improved estimators of the parameters ω, α and β and improved forecasts as in [7] and [16]. Therefore, the forecasting performance of Reg-ARMA-ARCH model is investigated. We assume to have knowledge of history up to the end of June 2010 and try to assess the performance ability of the model. In other words, we use daily data from 01/01/2008 until 30/06/2010 as a sort of "training data set" and measure the forecasting performance of the model until the end of 2010. Estimates for two zones (Sicily and North) are reported in Table 1. From these, we can confirm that seasonality is important also in returns, as well as calendar effects and intradaily information is always significant with the right signs. A "rolling windows" procedure has been used

	Sicily		North	
Variable	Coefficient	P-value	Coefficient	P-value
c	-0.128	0.000	-0.082	0.000
D_1	0.353	0.000	0.235	0.000
D_2	0.121	0.000	0.139	0.000
D_3	0.161	0.000	0.109	0.000
D_4	0.162	0.000	0.093	0.000
D_5	0.160	0.000	0.091	0.000
D_6	0.114	0.000	0.053	0.001
CE	-0.048	0.008	-0.053	0.000
Vol_{t-1}	0.000	0.097	0.000	0.065
r_{t-1}	0.726	0.000	0.786	0.000
ε_{t-1}	-1.191	0.000	-1.269	0.000
ε_{t-2}	0.180	0.023	0.176	0.034
ε_{t-3}	0.041	0.456	0.144	0.007
ε_{t-4}	-0.001	0.984	-0.050	0.232
ε_{t-5}	0.046	0.323	0.026	0.552
ε_{t-6}	0.027	0.530	0.042	0.289
ε_{t-7}	-0.062	0.055	-0.040	0.138
ω	0.078	0.000	0.051	0.000
α	0.132	0.0013	0.117	0.001
Vol_{t-1}	-1.15E-06	0.000	-8.85E-08	0.000
RV_{t-1}	0.137	0.000	0.124	0.000

Table 1: Estimates of Reg–ARMA–ARCH models for Sicily and North

to evaluate the out-of-sample forecasting performance of the models. In simple words, the whole time period is divided in two sub-periods, the first going from $t = 1$ to $t = T - m$ and the second covering the period from $t = T - m + 1$ to T. The procedure is iterative as we use a different set of information for estimating purposes rolling a windows of $T - m$ observations over the original data–set. Every time the estimated parameters are used to get a one–step–ahead forecast. In details:

- at time $T - m$ the vector of estimates θ_{T-m} is obtained through different models (RW1, RW7, Basic, Intermediate and Final) using data for $t = 1, \ldots, T - m$; the forecast in $T - m + h$, is then given by $y_{T-m+h|T-m} = f(\theta_{T-m}, y_{T-m})$;
- at time $T - m + 1$ the forecast for time $T - m + h + 1$ is obtained on data for $t = 2, \ldots, T - m + 1$, that is $y_{T-m+h+1|T-m+1} = f(\theta_{T-m+1}, y_{T-m+1})$.

- ...;
- the last forecast is estimated at time $T-h$ as $y_{T|T-h} = f(\theta_{T-h}, y_{T-h})$, using data for $t = d, \ldots, T-h$.

	Returns				
	RW1	RW7	Basic	Intermediate	Final
RMSE	0.188	0.144	0.095	0.095	0.096
MAPE	0.002	0.002	0.001	0.001	0.001
Theil's U	-	0.748	0.474	0.480	0.485
DM	-	1.535	3.311	0.150	-0.177
p-value	-	0.062	0.000	0.440	0.570
	Intraday volatility (1-step-ahead prediction)				
	RW1	RW7	Basic	Intermediate	Final
RMSE	0.013	0.015	0.013	0.013	0.012
MAPE	0.000	0.000	0.000	0.000	0.000
Theil's U	-	1.184	0.945	0.946	0.871
DM	-	-0.001	0.001	-1.862	1.862
p-value	-	0.500	0.499	0.969	0.031
	Standardized returns (1-step-ahead prediction)				
	RW1	RW7	Basic	Intermediate	Final
RMSE	1.429	1.134	0.876	0.848	0.793
MAPE	0.016	0.013	0.010	0.010	0.010
Theil's U	-	0.769	0.602	0.582	0.542
DM	-	1.245	1.580	1.975	2.088
p-value	-	0.107	0.057	0.024	0.018

Table 2: Forecasting performance in Sicily

At the end of the iterative procedure, $m-h+1$ h-step ahead forecasts are obtained. Analyzing daily data of electricity prices, if $m = 180$, we can check the h-day ahead forecasting performance for the last six months of data. We have measured the forecasting performance in predicting three variables: returns (r_t), conditional variance (σ_t^2) and standardized returns (r_t/σ_t) for Sicily and North.

To evaluate the gain obtained by using realized measures of volatility we considered four benchmark models: a simple random walk (RW1), a weekly random walk (RW7), the ARMA(1,7)–GARCH(1,1) model without regressors (form now on, called "Basic Model") and the Reg–ARMA(1,7)–GARCH(1,1) model, that is the previous one with regressors (form now on, called "Intermediate Model"). The RW1 is a classical benchmark model whose forecasts are commonly called "naive" predictions. The forecast function of the random walk is $y_{t|t-1} = y_{t-1}$, that is the observed average price of yesterday is the forecast for today. We take the value of two days ago if there was a holiday yesterday. The number of days in the past is increased accordingly when there are two or more contiguous holiday days. The RW7 is a forecast method which has been used as benchmark model in previous papers on electricity loads forecasting (see [6]). The forecast function for the RW7 is $y_{t|t-7} = y_{t-7}$, that is the average price observed one week ago is the forecast for today.

We use the root mean squared forecast error (RMSE), the mean absolute percentage forecast error (MAPE), the Theil's U index as a set measures to assess the predictive goodness of each model. We also apply the Diebold–Mariano test (see [5]) to compare different estimated models. Only results for Sicily have been reported in Table 2.

Conclusions

In this paper, intra-daily volatility information has been used to improve fitting and forecasting performance of conditional volatility models for electricity prices. The proposed models for the process of electricity volatility, enriched by intra-daily information (daily volumes and realized variance), reduce volatility persistence and provide good day–ahead forecasts. In details, looking at RMSE and MAPE, we can conclude that basic, intermediate and final models show forecasting performances better than the naive models. All proposed models produce similar forecasting errors, but lower than those of the first two models; and this for all three variables. In addition, it is evident that the final model exhibits lower forecasting errors for intraday volatility and standardized returns, at least in Sicily. Finally, considering the DM test, the forecasting performance for returns is deeply improved when implementing models more sophisticated than naive ones. Further developments of this work include the application of the same model to different markets in order to state a new stylized fact for electricity prices. The next theoretical step will be the introduction of an equation to model the temporal evolution of the realized volatility.

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