On Wishart ratios with dependent structure

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In multivariate statistics, the problems of interest are random covariance matrices (known as Wishart matrices) and ratios of Wishart matrices that arise in multivariate analysis of variance (see Morrison, 2005). As background, let us consider the Wishart ratio:

$$\boldsymbol{U} = (\boldsymbol{H}+\boldsymbol{E})^{-\frac{1}{2}}\boldsymbol{H}(\boldsymbol{H}+\boldsymbol{E})^{-\frac{1}{2}}$$

($\boldsymbol{H} \sim W_p(n,\boldsymbol{\Sigma})$ independent of $\boldsymbol{E} \sim W_p(m,\boldsymbol{\Sigma})$), this ratio is the genesis of the *matrix variate beta type I distribution* denoted as $\boldsymbol{U} \sim B_p^I(n,m)$.[1]

Let $\boldsymbol{H}_i \sim W_p(n_i,\boldsymbol{\Sigma})$, $i=1,2$, and independent of $\boldsymbol{E} \sim W_p(m,\boldsymbol{\Sigma})$, where

$$\boldsymbol{W}_i = (\boldsymbol{H}_1+\boldsymbol{H}_2+\boldsymbol{E})^{-\frac{1}{2}}\boldsymbol{H}_i(\boldsymbol{H}_1+\boldsymbol{H}_2+\boldsymbol{E})^{-\frac{1}{2}},\ i=1,2,$$

then it is evident that $\boldsymbol{W}_1 \sim B_p^I(n_1,n_2+m)$ and $\boldsymbol{W}_2 \sim B_p^I(n_2,n_1+m)$. However, they are correlated with a common "denominator" and the distribution of $\mathbb{W} = (\boldsymbol{W}_1 : \boldsymbol{W}_2)'$ is termed the *bimatrix variate beta type I distribution.* The corresponding Dirichlet distribution, that is for $i=1,\ldots,r$, was derived by Olkin and Rubin (1964).

The distribution of $\mathbb{U} = (\boldsymbol{U}_1 : \boldsymbol{U}_2)'$, where

$$\boldsymbol{U}_i = (\boldsymbol{H}_i+\boldsymbol{E})^{-\frac{1}{2}}\boldsymbol{H}_i(\boldsymbol{H}_i+\boldsymbol{E})^{-\frac{1}{2}}, i=1,2,$$

($\boldsymbol{H}_i \sim W_p(n_i,\boldsymbol{\Sigma})$, $i=1,2$, independent of $\boldsymbol{E} \sim W_p(m,\boldsymbol{\Sigma})$) has been independently studied by Bekker, Roux, Ehlers and Arashi (2011), Díaz-García and Jáimez (2010a) and Gupta and Nagar (2009). Bekker, Roux, Ehlers and Arashi (2011) referred to this distribution of $\mathbb{U}$ as the *bimatrix variate beta type IV distribution.*

For a detailed discussion on bimatrix variate beta distributions with bounded domain, the reader is referred to Ehlers (2011). The refreshing contributions made by Díaz-García and Jáimez (2010a, 2010b, 2011) should also be acknowledged.

In this paper, the main focus is to add a further independent Wishart random variate to the "denominator" of one of the ratios of the bimatrix variate beta type IV distribution; this results in deriving the exact expression for the density function of the *bimatrix variate extended beta type*

[1] There exist other definitions for the beta matrix $\boldsymbol{U}$, see Diaz-Garcia and Jaimez, 2010a.

IV distribution. The latter distribution leads to the proposal of the *bimatrix variate extended F* distribution. These two distributions set the platform for studying some interesting characteristics of these distributions. Armed with these results, the following are proposed: (i) the distribution of the ratio of the determinants of the components of the bimatrix variate extended beta type IV distribution; (ii) the distribution of the trace of the sum of the components of the bimatrix variate extended F distribution, as well as the bimatrix variate Kummer extended beta type IV distribution.

Lastly, we shift attention to the bivariate case (X_1, X_2) (see also El Bassiouny and Jones, 2009). This bivariate distribution can be expressed as an infinite mixture of the popular Jones' bivariate beta distribution (which was independently proposed by Jones (2001) and Olkin and Liu (2003)). It is well known that the stress-strength model describes the life of a component which has a random strength X_2 and is subjected to random stress X_1. The component fails if the stress (X_1) applied to it exceeds the strength (X_2) and the component will function satisfactorily whenever $P(X_1 < X_2)$. Therefore the distribution of X_1/X_2 receives attention, where (X_1, X_2) has this bivariate extended beta type IV distribution, together with some graphs and percentage points. This study is concluded with a few open problems that need to be addressed in future research.

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