K sample Cramér-von Mises tests for grouped data.

Zheng Sun

1. INTRODUCTION

Single sample tests for continuous data based on Cramér-von Mises statistics, for example W^2 and A^2, were well established before the 1960s. Watson (1961, 1962) introduced U^2, adopted from W^2, for testing fit when the observations are taken on the circumference of a circle. Kiefer (1959), Maag (1965) and Scholz and Stephens (1987) extended W^2, U^2 and A^2 to give K sample tests.

However, in much applied work, the data is grouped into cells, sometimes because of the difficulty of accurate measurement. Some examples from the literature are: In scour and fill studies, the measurement of the depth of the scour can be given to within 4 inches Spinelli (2001). In Brown (1994), birth counts in Australia are given as monthly data. Other examples can be found in the literature.

For single samples of grouped data, Choulakian, Lockhart and Stephens (1994) developed tests of Cramér-von Mises type when cell probabilities are known. Spinelli, Lockhart and Stephens (2007) extended this work to the case where parameters must be estimated. An important difference between the Cramer-von Mises and Pearson's X^2 is that the former take into account the order of the cells.

2. CRAMÉR-VON MISES TESTS

This article is a summary of Sun, Lockhart and Stephens (2011) who extended W^2, A^2 and U^2 to test that K samples have the same distribution for grouped data. For K samples, the three statistics are defined as follows: let n_i be the number of observations of the ith sample and $N = \sum_{i=1}^{K} n_i$ is the total sample size. Suppose the data for the ith sample are recorded as counts $o_{ij}, i = 1, \cdots, K; j = 1, \cdots, m$ in m cells whose lengths may be different, and suppose p_j is the probability that an original observation falls into cell j. Define $Z_j = \sum_{i=1}^{j}(S_i - T_i)$ where $S_{ij} = \sum_{l=1}^{j} o_{il}$ and $T_{ij} = \sum_{l=1}^{j} n_i p_{il}$, $\bar{Z}_i = \sum_{j=1}^{m} Z_{ij} p_j$ and $H_j = \sum_{l=1}^{j} p_l, j = 1, \ldots, m$.

The discrete K sample version of W^2, U^2 and A^2 are then defined by

$$W_K^2 = \sum_{i=1}^{K} n_i \sum_{j=1}^{m} Z_{ij}^2 p_j, \tag{1}$$

$$U_K^2 = \sum_{i=1}^{K} n_i \sum_{j=1}^{m} (Z_{ij} - \bar{Z}_i)^2 p_j, \tag{2}$$

$$A_K^2 = \sum_{i=1}^{K} n_i \sum_{j=1}^{K} Z_j^2 p_j / \{H_j(1 - H_j)\}. \tag{3}$$

Note that since $Z_{im} = 0$ and $H_m = 1$ the last term of A^2 is 0/0 and is set to 0.

For a given sample, one can estimate the p_j using

$$\hat{p}_j = \frac{\sum_{i=1}^{K} o_{ij}}{N}. \tag{4}$$

3. SUMMARY

In Sun, Lockhart and Stephens (2011), these tests are discussed. The following points are covered in some detail:

(1) Calculation of asymptotic points. For the continuous case, Kiefer (1959) showed that for K sample, the asymptotic points for unknown common c.d.f. can be found from assuming the estimated c.d.f. is correct and calculating the points for $K-1$ samples. Monte Carlo studies show that a similar result obtains for grouped data. This makes the asymptotic points easier to calculate.

(2)The asymptotic points are excellent approximation for finite samples and may be used to give tests with good level.

(3) Suggested corrections Brown (1982, 1994) for grouped data to enable continuous asymptotic distributions to be used are examined.

(4) Power studies are given which show that Cramér von-Mises statistics calculated as above give greater power than the statistics with group corrections. They also give greater power than X^2, and also than a Kolmogorov-Smirnov type modification for grouped data.

(5) Several examples from the literature are used to illustrate the tests.

(6) Programs written in R are available from the author to make all the calculations.

REFERENCES

B. M. Brown (1982). Cramer-von Mises distributions and permutation tests. *Biometrika*, 69, 619-624.

B. M. Brown (1994). Grouping corrections for circular goodness-of-fit tests. *Journal of the Royal Statistical Society*, 56, 275-283.

V. Choulakian, R. A. Lockhart & M. A. Stephens (1994). Cramér-von Mises statistics for discrete distributions, *The Canadian Journal of Statistics*, 22, 125–137.

J. C. Kiefer (1959). K- sample analogue of the Kolmogorov-Smirnov and Cramer-von Mises tests. *Ann. Math. Statist.*, 30, 420-47.

R. A. Lockhart & J. J. Spinelli & M. A. Stephens (2007). Cramér-von Mises statistics for discrete distributions with unknown parameters. *Canadian Journal of Statistics*, 35, 125–133(9).

U. R. Maag (1966). A k sample analogue of Watson's U^2 statistic. *Biometrika*, 53, 579-583.

J. J. Spinelli (2001). Testing fit for the grouped exponential distribution. *Canadian Journal of Statistics*, 29, 451–458(3).

Z. Sun, R. A. Lockhart & M. A. Stephens (2011). K sample EDF tests for grouped data and some comparisons. *Research report, Statistics and Actuarial Science, Simon Fraser University, Canada.*

G. S. Watson (1961). Goodness-of-fit tests on a circle. I. *Biometrika*, 48, 109-14.

G. S. Watson (1962). Goodness-of-fit tests on a circle. II. *Biometrika*, 49, 57-63.