Volatility of High-Frequency Returns on Foreign Exchange and Stable Innovations

Oral, Ece[1]
Central Bank of the Republic of Turkey, Research and Monetary Policy Department
İstiklal Cad. 10 Ulus
Ankara, 06100, Turkey
E-mail: Ece.Oral@tcmb.gov.tr

Oral, Evrim
LSUHSC, School of Public Health
1615 Poydras Street, Suite 1400
New Orleans, LA 70112, USA
E-mail: eoral@lsuhsc.edu

Abstract

Many models in finance are often based on the assumption that the random variables follow a Gaussian distribution. It is now well known that empirical data have frequently occurring extreme values and cannot be modeled with the Gaussian distribution. The stable distributions, a class of probability distributions that allow skewness and heavy tails, have received great interest in the last decade because of their success in modeling financial data that depart from the Gaussian distribution. This study examines the statistical distributions of intra-daily TRY/USD foreign exchange changes. The volatility of the return series are calculated using the Stable GARCH models. It is found that the GARCH model with stable innovations fits returns better than the Normal distribution.

Keywords Stable distribution; Parameter estimation; high-frequency data, Foreign exchange, Conditional heteroskedasticity

JEL Classification C13; C16; C50; F31

1. Introduction

Modern finance relies heavily on the assumption that the random variables under investigation follow a Normal distribution (Rachev and Mittnik, 2000). However, finance data often depart from the Normal model, in that their marginal distributions are heavy-tailed. The use of stable distributions to finance has been introduced via the works of Mandelbrot (1963) and Fama (1965). The peaked and heavy-tailed nature of the return distribution led the authors to reject the standard hypothesis of normally distributed returns in favor of the stable distribution. Since then, the stable distribution has been used to model both the unconditional and conditional return distributions (Marinelli et.al, 2001).

While different heavy-tailed distributions such as GED or Student's t can be used for modeling financial variables, stable distributions are preferred due to the generalized Central Limit Theorem. According to this theorem, regardless of the existence of the variance, the limiting distribution of a sum of independent and identically distributed random variables is stable (Borak, Härdle and Weron, 2005). Additionally, stable distributions are a rich class of probability distributions that allow skewness and heavy tails. Thus, they are widely used in modeling heavy tailed data (Zolotarev, 1986).

The conditional distribution of assets returns in GARCH models is assumed to be Normal. However,

[1] The views expressed are those of the author and should not be attributed to the Central Bank of the Republic of Turkey.

this model specification is not proper for many financial time series because of the leptokurtic behavior of the data. Therefore, the distributions such as the Student's t distribution, GED, and the Laplace distribution have been suggested to be the distributional models for innovations. Besides these models, GARCH models with stable Paretian innovations in financial modeling has been recently used in the literature because of allowing skewness and leptokurtosis of financial returns (Curto et al., 2009).

This paper examines the statistical properties of the intraday returns and the usage of stable GARCH models for modeling high-frequency data. It also shows that this model is more suitable than Normal models.

The remainder is organized as follows. Next section discusses GARCH model with stable innovations. Section 3 explains the statistical properties of returns and presents the initial findings. It also discusses the estimation results and compares the goodness-of-fit of the two conditional distributions. Section 4 summarizes the concluding remarks.

2. Stable distributions:

Stable distributions do not have an analytic closed form but can be expressed by their characteristic function,

$$\phi_X(t) = E\left(e^{itX}\right) = \exp\left(i\delta t - |ct|^{\alpha}\left[1 + i\beta \,\mathrm{sgn}(t)\, w(t,\alpha)\right]\right) \tag{1}$$

where

$$w(t,\alpha) = \begin{cases} -\tan\left(\dfrac{\pi\alpha}{2}\right) & , \quad \alpha \neq 1 \\ (2/\pi)\ln|t| & , \quad \alpha = 1 \end{cases}$$

$-\infty<t<\infty$, $0<\alpha\leq 2$, $|\beta|\leq\min(\alpha, 1-\alpha)$, $c>0$, $-\infty<\delta<\infty$.

A stable distribution has four parameters; α, β, δ and γ ($\gamma=c^{\alpha}$). α is called characteristic exponent and interpreted as a shape parameter. The Normal distribution is stable with $\alpha=2$ and is the only stable distribution which second and higher absolute moments exist. When $\alpha<2$, absolute moments of order equal to and greater than α do not exist while those of order less than α do. The distribution becomes heavy tailed. The tail thickness increases as α decreases. δ and c are the location and scale parameters respectively. When β (skewness parameter) is positive (negative), the distribution is skewed to the right (left). If β is zero, the distribution becomes symmetric about δ (location parameter). As α approaches to 2, the distribution approaches to a Normal distribution regardless of β (Fama and Roll, 1968).

Following Curto et al. (2009), let the time series y_t be given as ARMA(p,q) process:

$$y_t = \mu + \sum_{i=1}^{p} \gamma_i y_{t-i} + \sum_{i=1}^{q} \xi_j u_{t-j} + u_t \tag{2}$$

We express the conditional standard equation of a Stable GARCH (GARCH(r;s)) as[2]:

$$\sigma_t = \theta_0 + \sum_{i=1}^{r} \theta_i |u_{t-i}| + \sum_{j=1}^{s} \phi_j \sigma_{t-j} \tag{3}$$

where $u_t = \sigma_t\varepsilon_t$ and $\varepsilon_t \overset{iid}{\sim} S_{\alpha,\beta}(0,1)$. $S_{\alpha,\beta}(0,1)$ denotes the standard asymmetric Stable Paretian distribution

---

[2] A Gaussian (distribution) GARCH model is given: $\sigma_t^2 = \theta_0 + \sum_{i=1}^{r} \theta_i u_{t-i}^2 + \sum_{j=1}^{s} \phi_j \sigma_{t-j}^2$

with location parameter (δ =0), and unit scale parameter (c=1)[3]. The probability density and the likelihood functions of the Stable distribution GARCH are nontrivial, and a Fast Fourier Transforms (FFT) procedure is employed for the maximum likelihood estimation (MLE) algorithm. We follow the MLE procedure of conditional heteroskedasticity models with Stable distributions presented by Mittnik et al. (1999)[4].

To examine the intraday returns, assume the mean equation as follows:

$$y_t = \mu + u_t \tag{4}$$

where y_t is the returns and $u_t = \sigma_t \varepsilon_t$, $\varepsilon_t \overset{iid}{\sim} S_{\alpha,\beta}(0,1)$.

We also impose certain stability conditions to estimate the Stable GARCH models (Rachev and Mittnik, 2000).

3. Empirical Results:

The data consist of the TRY/USD exchange rate realized at Turkish Interbank Foreign Exchange Market at 10:30, 11:30, 12:30, 13:30, 14:30 and 15:30 starting from April 1st 2002 until May 13th 2010. At the indicated hours, the TRY/USD rate is the average value of the averages of the buying and selling rates as quoted by banks in the Interbank Foreign Exchange Market for 1 USD. The holding periods are one-hour, five-hour, close to open (19-hour), open to open (24-hour), close to close (24-hour) and average to average (24-hour) changes. The logarithmic returns are calculated for each holding period and the summary statistics related to each period can be seen in Table 1.

(Table 1)

The values of kurtoses are above the normal value of 3.0 for every holding period that points out the heavy-tailed behavior of high-frequency foreign exchange changes. According to Jarque-Bera statistics, all variables indicate non-normal distributions.

(Figure 1)

According to Figure 1, it can be said that stable distribution fits better than Normal distribution for both series.

(Table 2)

Table 2 presents Normal and Stable GARCH as benchmark models for different holding periods. The results show that GARCH parameters are significant in both models for each holding period. The Stable GARCH models indicate that the shape parameters are significant and both are less than 2 based on t-tests indicating heavy tailed pattern, and the skewness parameters are positive and significant. Log likelihood, AIC and SBC are the criterions that are used to decide on which model will be the final one. The Stable GARCH models show significantly higher log likelihood values compared to the Normal GARCH models and yield much smaller values of AIC and SBC. They are preferable models based on the goodness of fits. These results are no surprise since the Stable models take into account the non-normal distribution of the time series estimated.

4. Conclusions:

It is of great importance for those in charge of managing risk to understand how financial asset returns are distributed. Empirical evidence has led many practitioners to reject the normality assumption supporting various heavy-tailed alternatives and it is now commonly accepted that financial asset returns are, in fact, heavy-tailed.

Stable distributions are the probability distributions that allow skewness and heavy tails. Therefore,

---

[3] These assumptions simplify the estimation, but will not alter the properties of the stable distribution. See, for example, Curto et al. (2009) and Tavares et al. (2008).

[4] We thank Professor Curto for providing us Matlab codes to estimate the model.

they are widely used in modeling heavy tailed data (Zolotarev, 1986). Recent studies show that stable distributions have been used for modeling stock returns, foreign exchange rate changes, commodity-price movements, and real estate returns (McCulloch, 1997).

This paper examines the statistical distributions of high frequency (intra-daily) TRY/USD foreign exchange changes and the volatility of the return series by employing the Stable GARCH models.

The return series are gathered for the time period :April 1st 2002-May 13th 2010. The distributions of the returns are examined. The empirical results show that the foreign exchange changes do not follow the normal distribution and show heavy-tailed behaviors.

In order to examine the volatility, Normal and Stable GARCH as benchmark models are constructed for different holding periods. The results indicate that GARCH parameters are significant in both models for each holding period. The goodness of fit supports the use of Stable GARCH over Normal GARCH specifications. The Stable GARCH models indicate that the shape parameters are significant and both are less than 2 based on t-tests indicating heavy tailed pattern, and the skewness parameters are positive and significant. Another important result is that, when the holding period increases the shape and skewness parameters increase for both periods.

On the whole, the estimated parameters suggest that the normality restriction is misleading and, thus, imposing the normality may cause a bias for financial modeling. The stable distribution is relatively effective in capturing large changes in exchange rate movements, while the normal distribution screens out the outliers.

Table 1: Summary Statistics of Returns

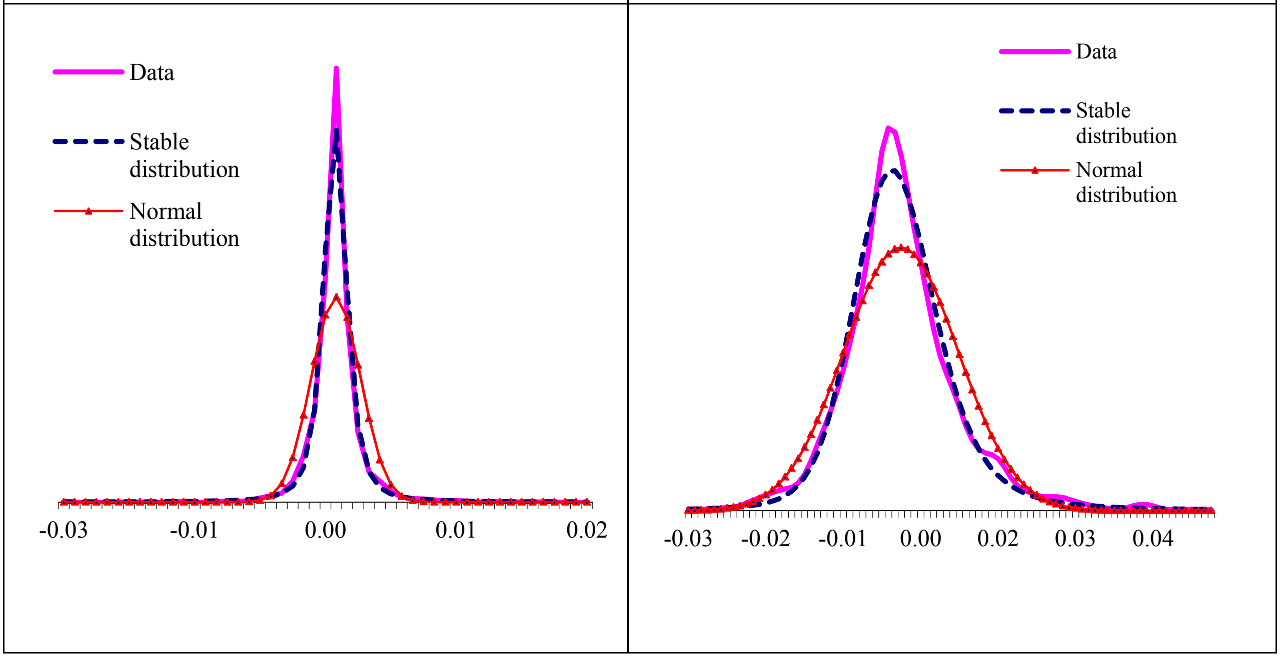
	One-Hour	**Five-Hour**	**Close to Open (19-hour)**	**Close to Close (24-hour)**	**Open to Open (24-hour)**	**Average to Average (24-hour)**
# Obs.	10142	2028	2027	2026	2028	2028
Mean	-0.00001	-0.00004	0.00010	0.00006	0.00006	0.00006
Median	-0.00005	-0.00023	-0.00043	-0.00071	-0.00049	-0.00074
Minimum	-0.02494	-0.03732	-0.12560	-0.09516	-0.14528	-0.11932
Maximum	0.02468	0.03810	0.06760	0.05728	0.07990	0.07043
Std.Dev.	0.00237	0.00549	0.00881	0.00980	0.01091	0.00961
Skewness	0.61	0.41	-0.49	0.37	-0.40	-0.04
Kurtosis	15.43	8.79	28.47	11.09	23.12	18.78
Jarque-Bera	65961.19*	2885.33*	54858.23*	5563.33*	34258.69*	21052.69*

Table 2: Maximum likelihood estimates and goodness-of-fit statistics of GARCH(1,1) for different holding periods (standard errors are in parentheses, t-statistics are in square brackets)

Estimates	**One-Hour Changes**		**Five-Hour Changes**		**19-Hour Changes**		**24-Hour Changes**	
	Normal	**Stable**	**Normal**	**Stable**	**Normal**	**Stable**	**Normal**	**Stable**
Intercept (μ)	-0.0000582 (1.43E-05) [-4.08]	0.0000001 (1.72E-05) [0.01]	-0.0002260 (8.36E-05) [-2.70]	0.0000001 (9.61E-05) [0.00]	-0.0000096 (1.30E-04) [-0.07]	0.0001517 (1.38E-04) [1.10]	-0.0002630 (1.65E-04) [-1.59]	0.0000001 (1.60E-04) [0.00]
θ_0	0.0000001 (2.32E-09) [36.34]	0.0000264 (2.86E-06) [9.24]	0.0000011 (1.05E-07) [10.59]	0.0001123 (2.62E-05) [4.29]	0.0000029 (2.78E-07) [10.46]	0.0002664 (5.00E-05) [5.33]	0.0000024 (3.71E-07) [6.50]	0.0002685 (5.13E-05) [5.23]
θ_1	0.1103390 (2.58E-03) [42.81]	0.0681020 (3.82E-03) [17.84]	0.1650930 (1.14E-02) [14.47]	0.0879770 (1.01E-02) [8.67]	0.1878870 (1.53E-02) [12.28]	0.1233400 (1.28E-02) [9.67]	0.1493090 (1.21E-02) [12.33]	0.1003800 (1.08E-02) [9.30]
ϕ_1	0.8839660 (2.23E-03) [396.30]	0.8838500 (6.57E-03) [134.60]	0.8104060 (1.05E-02) [77.06]	0.8499600 (1.80E-02) [47.15]	0.7866550 (1.19E-02) [66.29]	0.7896500 (2.16E-02) [36.62]	0.8337950 (1.04E-02) [80.19]	0.8288000 (1.88E-02) [44.17]
α	- -	1.600 (1.77E-02) [90.45]	- -	1.705 (3.79E-02) [45.02]	- -	1.778 (3.39E-02) [52.49]	- -	1.803 (3.42E-02) [52.73]
β	- -	0.118 (3.29E-02) [3.58]	- -	0.322 (9.02E-02) [3.57]	- -	0.543 (1.03E-01) [5.27]	- -	0.662 (1.18E-01) [5.59]
Log Likelihood	48874	49984	7950	8069	7108	7223	6809	6868
AIC	-97739	-99967	-15892	-16136	-14207	-14445	-13609	-13734
SBC	-97710	-99969	-15869	-16138	-14185	-14447	-13587	-13736

Figure 1: Distribution of Return Series

a. One-Hour Changes | ***b. Close to Close Changes***



REFERENCES

Borak, S., Hardle, W., Weron, R., 2005. Stable Distributions. SFB 649 Discussion Paper 2005-008, Humboldt-Universität zu Berlin.

Curto, J. D., Pinto J. C. and Tavares G. N., 2009. Modeling stock markets' volatility using GARCH models with Normal, Student's t and stable Paretian distributions, Statistical Papers 50, 311-321.

Fama, E.F., 1965. The behaviour of stock market prices. Journal of Business 38:34-105.

Fama, E.F., Roll, R., 1968. Some Properties of Symmetric Stable Distributions. Journal of the American Statistical Association 63:817-835.

Mandelbrot, B., 1963, New methods in statistical economics, Journal of Political Economy, LXXI, 421-440.

Marinelli, C. Rachev, S.T., Roll, R., 2001. Subordinated Exchange Rate Models: Evidence for Heavy Tailed Distributions and Long-Range Dependence. Mathematical and Computer Modelling, 34:955-1001.

McCulloch, J.H., 1997. Tail Thickness to Estimate the Stable Index □: A critique. Journal of Business & Economic Statistics 15:74-81.

Mittnik, S., Rachev, S. T., Doganoglu, T. and Chenyao, D., 1999. Maximum likelihood estimation of stable Paretian models, Mathematical and Computer Modeling, 29, 275-293.

Rachev, S., Mittnik, S., 2000. Stable Paretian Models in Finance. Series in Financial Economics and Quantitative Analysis. John Wiley & Sons Ltd.

Tavares, A. B., Curto, J.D., and Tavares, G.N, 2008. Modeling heavy tail and asymmetry using ARCH-type models with stable Paretian distribution, Nonlinear Dynamics 51, 231-243.

Zolotarev, V.M., 1986. One-Dimensional Stable Distributions. Vol 65 of Translations of Mathematical Monographs, American Mathematical Society, Providence, Rhode Island.