Monitoring Validity of Daily Volatility Model

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1 Introduction

Forecasting daily volatility of risky assets is of importance for numerous financial applications. The availability of intraday ultra high frequency price observations opens new horizons in volatility modeling. The realized volatility measures (cf. Andersen and Bollerslev 1998, Barndorff-Nielsen and Shephard 2004) exploit high frequency returns in order to provide precise estimators of the daily integrated volatility. There are numerous approaches for modeling and forecasting daily volatility, which are based on the realized volatility measures. Such popular approaches as ARFIMA, AR models of higher order or MIDAS (Ghysels, Santa-Clara and Valkanov, 2006) are able to reflect the complex volatility dynamics and to account for the empirical stylized facts characterizing daily volatility series.

We concentrate, however, on a rather simple model for daily volatility in spirit of Fleming and Kirby (2003). The volatility process is assumed to follow a linear state space representation. The observation equation describes relation between a volatility measure and the true (unobservable) integrated volatility, whereas the state equation presumes an AR(1) dynamics. This simple model should provide short-term volatility forecasts until it remains correct. Thus, it is required to check the validity of the model at each new time point. Control charts from statistical process control are appropriate statistical decision rules for this purpose (Montgomery, 2005). The control chart exploits the process of volatility forecasting errors, whereas a signal from the control chart indicates on a possibility that the initial assumptions concerning the process of interest are no longer satisfied.

2 Modeling of Integrated Volatility

Let the log asset price $y(t)$, $t \geq 0$ be the Brownian semimartingale plus a jump component $J(t)$

$$(1) \quad y(t) = \int_0^t \mu(u)\mathrm{d}u + \int_0^t \sigma(u)\mathrm{d}W(u) + J(t),$$

where $\mu(\cdot)$ is a locally bounded predictable drift process, $\sigma(\cdot)$ is a strictly positive spot volatility process, and $W(\cdot)$ is a standard Brownian motion. The processes $\mu(\cdot)$ and $\sigma(\cdot)$ are presumed to be stochastically independent of $W(\cdot)$. The component $J(t) = \sum_{i=1}^{N_t} \kappa_i$ corresponds to the jump process, with a simple counting process N_t, finite for all t, and nonzero random variables κ_i. In the case of no jumps, i.e. $N_t = 0$ for all t, the log asset price $y(t)$, $t \geq 0$ simplifies to the Brownian semimartingale. The integrated volatility (IV) $\sigma_t^2 = \int_{t-1}^t \sigma^2(u)\mathrm{d}u$ reflects the continuous part of the overall variability for the day t. This quantity plays the major role in the further presentation.

Since σ_t^2 is not observable it is necessary to estimate it. Here we consider bipower variation measure (BV) on intraday data proposed by Barndorff-Nielsen and Shephard (2004):

$$BV_t = \frac{\pi}{2}\frac{M}{M-1}\sum_{m=1}^{M-1}|R_{t,m}||R_{t,m+1}|,$$

with $R_{t,m} = y(t_m) - y(t_{m-1})$ for $t-1 = t_0 < t_1 < \cdots < t_M = t$.

The BV converges in distribution to the IV under certain regularity conditions (Barndorff-Nielsen and Shephard, 2004) with $\max_m(t_m - t_{m-1}) \to 0$, $M \to \infty$. Although the BV is not an efficient estimator, it is robust against jumps in the log-price behavior. Since the distribution of the log volatility is more symmetric, further we are modeling the log volatility. The log BV remains a consistent estimator of the log IV (cf. Bickel and Doksum 2001, p.461)

$$\frac{\log(BV_t) - \log(\sigma_t^2)}{v_t^{1/2}} \xrightarrow{\mathcal{L}} \mathcal{N}(0,1), \qquad \text{with} \quad \max_m(t_m - t_{m-1}) \to 0, \quad M \to \infty,$$

where v_t is the variance of the log BV. It can be consistently estimated on the intraday data as

$$(2) \quad \hat{v}_t = \left(\frac{\pi^2}{4} + \pi - 3\right)\frac{\pi^2}{4}\frac{M}{M-3}\sum_{m=1}^{M-3}|R_{t,m}||R_{t,m+1}||R_{t,m+2}||R_{t,m+3}| \Big/ BV_t^2.$$

Let the log IV follow the simple linear state space representation (cf. Fleming and Kirby, 2003):

$$(3) \quad \log(\sigma_{t+1}^2) - a = \phi\left[\log(\sigma_t^2) - a\right] + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0,q), \qquad |\phi| < 1,$$

$$(4) \quad \log(BV_t) = \log(\sigma_t^2) + \gamma_t, \qquad \gamma_t \sim \mathcal{N}(0, v_t).$$

The model has three parameters $\{a, \phi, q\}$. The variance v_t can be measured on intraday data by (2). The innovations $\{\varepsilon_t\}$ and $\{\gamma_t\}$ are assumed to be mutually and serially uncorrelated. The empirical (local) modeling advocates the AR(1) process in the state equation. The model (3)-(4) can be estimated by a maximum likelihood method. In general case the quasi maximum likelihood estimation procedure provides standard errors which are robust with respect to non-normal error components.

The best linear forecasts of $\log(\sigma_t^2)$ and $\log(BV_t)$ conditional on the information set $\mathcal{I}_{t-1}$ are their projections onto $\mathcal{I}_{t-1}$ (Hamilton 1994, p. 134), which can be calculated recurrently by

$$\log(\sigma_{t|t-1}^2) = a + \phi\left[\log(\sigma_{t-1|t-2}^2) - a\right] + \frac{\phi p_{t-1|t-2}}{p_{t-1|t-2} + v_{t-1}}\left[s_{t-1} - \log(\sigma_{t-1|t-2}^2)\right],$$

$$\log(BV_{t|t-1}) = \log(\sigma_{t|t-1}^2),$$

with $\log(BV_{1|0}) = \log(\sigma^2_{1|0}) = a$. The conditional variance $p_{t|t-1} = \text{var}\left[\log(\sigma^2_t) - \log(\sigma^2_{t|t-1})\right]$ is updated as

$$p_{t|t-1} = \phi^2 \frac{p_{t-1|t-2}}{p_{t-1|t-2} + v_{t-1}} v_{t-1} + q, \quad p_{1|0} = \frac{q}{1-\phi^2}.$$

The forecasting errors η_t are calculated as the difference between the observed $\log(BV_t)$ and its conditional forecast $\log(BV_{t|t-1})$:

$$\eta_t = \log(BV_t) - \log(BV_{t|t-1}).$$

The following proposition, proven by Golosnoy, Okhrin and Schmid (2011), established the stochastic processes of the forecasting errors under model validity. These properties are required for monitoring the validity of the state space representation in (3)-(4).

Proposition. Assuming (1) for the log price, (3)-(4) for the log volatility, the forecasting errors $\eta_t = s_t - s_{t|t-1}$ have the conditional expectation $E(\eta_t) = 0$ and variance $\text{var}(\eta_t) = p_{t|t-1} + v_t$ for all $t \in \mathbf{N}$, and are not autocorrelated. Moreover, assuming that ω_1, ε_t and γ_t are normally distributed for all t and uncorrelated, η_t is conditionally normally distributed $\eta_t \sim \mathcal{N}(0, p_{t|t-1} + v_t)$ for all t.

3 Sequential Monitoring of Model Validity

Statistical process control suggests control charts as suitable tools for monitoring stochastic properties of the process of interest. We exploit the standardized forecasting errors $X_t = \eta_t/(p_{t|t-1} + v_t)^{1/2}$ for our analysis. The model (3)-(4) given the parameters $\{a, \phi, q\}$ is assumed to be valid at $t = 0$. A control chart starts at $t = 1$ for making on-line decisions between the hypotheses on every new day $t \geq 1$

$$(5) \quad H_{0,t}: \ E(X_t) = 0 \qquad \text{vs.} \qquad H_{1,t}: \ E(X_t) \neq 0.$$

If $H_{0,t}$ remains valid for all $t \geq 1$ the monitored process is called to be in-control, otherwise out-of-control.

Each control chart consists of a control statistic Z_t, which depends on the process $\{X_t\}$, and a critical limit $c > 0$. A control chart gives a signal at t if $|Z_t| > c$. A good control chart should give seldom (false) signals in the in-control state and a correct signal immediately after the process gets out-of-control. The time period before the first signal is called the run length $L(c) = \inf\{t \geq 1 : \ |Z_t| > c\}$. The common criteria for the choice of c is to set the in-control average run length (ARL) equal to a (large) predetermined value $\mathcal{A}$, i.e. $E(L(c) \,|\, H_{0,t} \,\forall t) = \mathcal{A}$. Here the in-control ARL is set to be equal 120 which roughly correspond to half a year of daily observations.

There are various control charts suitable for making decisions between the hypotheses in (5). Here we make use of the Shewhart control chart with $Z_t = X_t$, which is a special case of EWMA control charts (Montgomery, 2005). It reacts fast on large changes in the monitored processes. A Monte Carlo simulation study investigates forecasting losses and control chart performance in different out-of-control situations. The detecting ability of the control chart is analyzed with the out-of-control ARLs. The out-of-control situations assume different types of changes in the model parameters $\{a, \phi, q\}$ with both normally and t-distributed innovations. Note that we change only one parameter each time, whereas the other two remain unchanged. The control limit is chosen $c = 2.638$ in order to provide the in-control ARL $\mathcal{A} = 120$ assuming normally distributed innovations. The pre-run initialization period consist of 100 observations.

Assume that the true model has changed (i.e. H_1 is valid), but the forecasts are still conducted with the H_0 model parameters. The corresponding forecasting errors serve for the calculation of the forecasting losses $L = T^{-1}\sum_{t=1}^{T} X_t^2$. Table 1 reports the average losses $\bar{L}$ which are calculated for 10^5 replications with both normally and t-distributed innovations. Table 2 presents the ARLs for both normally and t-distributed innovations calculated on 10^5 replications.

a_1	$\mathcal{N}$	t_8	ϕ_1	$\mathcal{N}$	t_8	q_1	$\mathcal{N}$	t_8
-1.00	2.68	2.86	0.1	1.04	1.39	0.134	0.73	0.97
-0.75	1.95	2.11	0.2	1.01	1.35	0.159	0.83	1.11
-0.50	1.42	1.60	0.3	1.00	1.33	0.184	0.93	1.25
-0.25	1.11	1.27	**0.4**	**1.00**	1.33	0.209	1.04	1.38
0.00	**1.00**	1.18	0.5	1.02	1.36	0.234	1.14	1.52
0.25	1.11	1.27	0.6	1.08	1.44	0.259	1.24	1.66
0.50	1.42	1.60	0.7	1.19	1.59	0.284	1.34	1.79
0.75	1.95	2.11	0.8	1.45	1.93	0.309	1.45	1.93
1.00	2.68	2.86	0.9	2.27	3.02	0.334	1.55	2.07

Table 1: Average losses $\bar{L}$ for normally and t-distributed innovations and different types of changes in the parameter vector. In-control values are $a = 0$, $\phi = 0.4$, $q = 0.2$.

a_1	$\mathcal{N}$	t_8	ϕ_1	$\mathcal{N}$	t_8	q_1	$\mathcal{N}$	t_8
-1.00	11.3	8.97	0.1	103.7	33.5	0.134	499.9	78.3
-0.75	21.0	14.1	0.2	114.7	35.0	0.159	263.3	55.4
-0.50	42.2	21.8	0.3	120.8	35.6	0.184	158.2	41.8
-0.25	84.5	30.9	**0.4**	**120**	35.5	0.209	104.8	33.0
0.00	**120**	35.8	0.5	110.2	34.2	0.234	74.5	26.9
0.25	84.5	31.0	0.6	92.2	31.6	0.259	56.0	22.6
0.50	42.1	21.9	0.7	68.4	27.5	0.284	44.0	19.5
0.75	21.0	14.1	0.8	44.6	22.3	0.309	35.6	16.9
1.00	11.3	8.95	0.9	26.5	16.6	0.334	29.6	15.1

Table 2: Simulated ARLs for normally distributed and t-distributed innovations and different types of changes in the parameter vector. In-control values are $a = 0$, $\phi = 0.4$, $q = 0.2$.

Table 2 confirms that changes causing the largest average forecasting losses, as shown in Table 1, could be detected quite quickly. The obtained results allow us to conclude that the Shewhart chart is suitable for detecting important changes in the volatility model parameters.

The empirical application in Golosnoy et al. (2011) illustrates this approach based on four highly liquid U.S. stocks. The in-control model is estimated based on the in-sample period, whereas the out-of-sample observations are exploited for sequential monitoring purposes. The in-sample results do not allow us to reject the state space volatility model for any of the four stocks. The out-of-sample monitoring clearly rejects the chosen state space representation for General Motors Company case because of many clustered signals, which can also be explained by the publicly available information.

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