



## Testing the rank of time-varying covariance matrices

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### Abstract:

We consider the instantaneous (or spot) covariance matrix  $\Sigma(t)$  of a multidimensional continuous-time process  $X(t)$ . The data is given by high-frequency observations of  $X$  on a fixed time interval. We test the null hypothesis  $H_0$  that  $\text{rank}(\Sigma(t)) \leq r$  for all  $t$  against the alternative  $\lambda_{r+1}(\Sigma(t)) \geq v_n$  that the  $(r+1)$ st eigenvalue is larger than some signal detection rate  $v_n$ , tending to zero with sample size  $n$ . This problem can be embedded in the classical nonparametric signal detection framework, but it has many unexpected features. For instance, the optimal detection rate  $v_n$  depends on a regularity assumption on  $\Sigma(t)$  under the null, not the alternative and a possible spectral gap leads to significantly better detection rates. The proofs rely on perturbation and deviation inequalities for random matrices which might have independent interest. Further results under observational noise will be discussed. The findings are illustrated throughout by applications to intraday data from government bonds.

### Keywords:

Principal Component Analysis, empirical covariance matrix, nonparametric signal detection, random matrix

### 1. Introduction:

<Introduction>

### 2. Methodology:

<Methodology>

### 3. Result:

<Result>

### 4. Discussion and Conclusion:

<Discussion and Conclusion>

### References:

- 1.
- 2.
- 3.

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